# 21 GEOMETRICAL CONSTRUCTIONS

## Definition of a geometric construction

**A geometric construction** is **an accurate** drawing of a plane figure.

For geometric constructions, the following will be needed: a pencil, a ruler, a pair of compasses, a protractor and a set square.

Before making any construction, it is helpful to make a **rough sketch** of what you are to draw. All the dimensions (measurements) should be shown on the sketch.

## Construction of triangles

A triangle can be constructed given measurements of:
(i) all the three sides,
(ii) two sides and one angle, or
(iii) one side and two angles.

### Construction of a triangle, given three sides

Using a ruler and a pair of compasses only, construct triangle PQR such that PQ = 2.5 cm, QR = 3.5 cm and PR = 5 cm.

*Procedure:*

(a) Draw a rough sketch of the triangle to be constructed and on it, indicate all the given measurements (Fig.21-1).

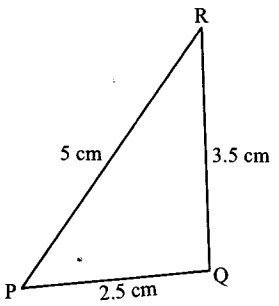



*Fig.21-1*

(b) Construct the triangle accurately using the following steps.

(i) Draw a line and mark a point P on it.

(ii) On the line, mark off a point Q, 2.5 cm from P, using a pair of compasses.

(iii) With P as the centre and radius 5 cm, draw an arc.

(iv) With Q as the centre and radius 3.5 cm, draw another arc to intersect the arc in (iii) at R.

(v) Join P to R, and Q to R. PQR is the required triangle (Fig.21-2).

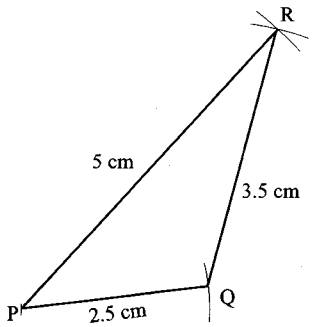
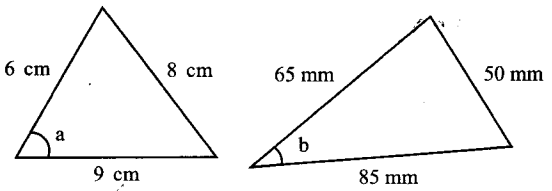



*Fig.21-2*

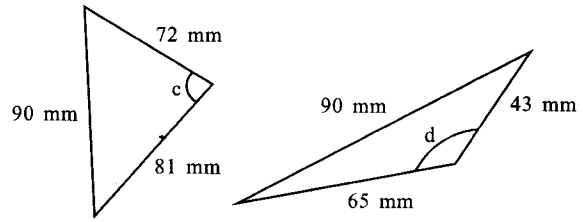
### Exercise 21.1

Use a ruler and a pair of compasses only in this exercise.

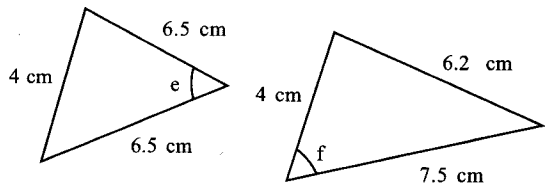
1. Construct the triangles whose sketches are given in Fig.21-3. Measure the marked angles.



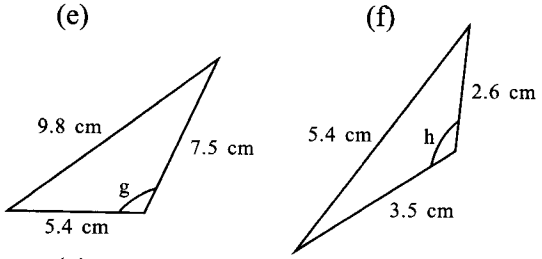

(a) (b)




(c) (d)




(e) (f)




(g) (h)

*Fig. 21-3*

**2.** Construct Δ ABC such that

(a) AB = 40 mm, BC = 50 mm, AC = 60 mm

(b) AB = 50 mm, BC = 85 mm, AC = 75 mm

(c) AB = 65 mm, BC = 45 mm, AC = 60 mm

(d) AB = 8.3 cm, BC = 8.3 cm, AC = 5 cm

(e) AB = 7 cm, BC = 6 cm, AC = 4 cm

(f) AB = 5.6 cm, BC = 5 cm, AC = 7.5 cm

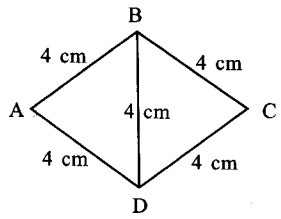
In each case measure ∠ ABC.

**3.** Make accurate constructions of the diagrams in Fig.21-4 and

(i) measure ∠ BCD in each case.

(ii) name the plane figures in (a) and (b).

(a)




(b)

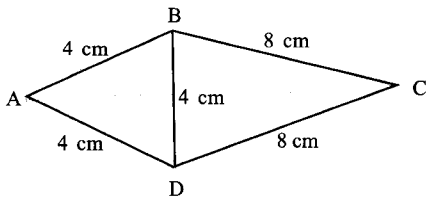



*Fig.21-4*

**4.** Try to construct triangles using the following measurements.

(a) 3 cm, 4 cm, 8 cm (b) 10 cm, 7 cm, 3 cm

(c) 5 cm, 5 cm, 12 cm

What do you notice in each case?

> Note that in order to be able to construct a triangle, the sum of the lengths of the two shorter sides must be greater than the length of the third side.

**5.** Make an accurate construction of the diagram in Fig.21-5. Measure ∠ BAC and ∠ ACD. What can you say about lines AB and DC?

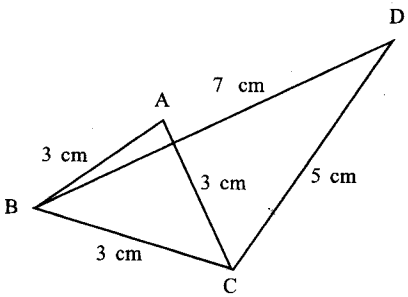



*Fig.21-5*

## Construction of a triangle, given two sides and an angle

Construct Δ ABC such that ∠ ABC = 50° , AB = 4 cm, BC = 6 cm.

*Procedure:*

(a) First make a rough sketch (Fig.21-6).

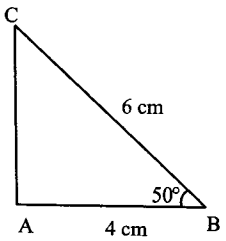



*Fig.21-6*

(b) Construct Δ ABC as follows.

(i) Draw a line and mark a point A on it.

(ii) On the line, mark off a point B, 4 cm from A, using a pair of compasses.

(iii) At point B, use your protractor to measure an angle of 50° and draw the line BX.

(iv) Mark a point C on line BX sucH that BC = 6 cm.

(v) Join AC to complete Δ ABC (Fig. 21-7).

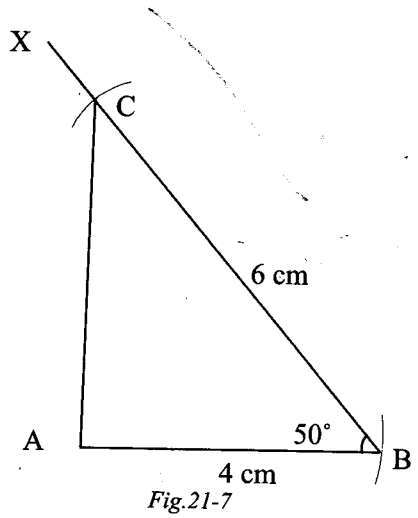



*Fig.21-7*

## Exercise 21.2

**1.** Construct the triangles sketched in Fig. 21-8 and measure the sides marked with letters.

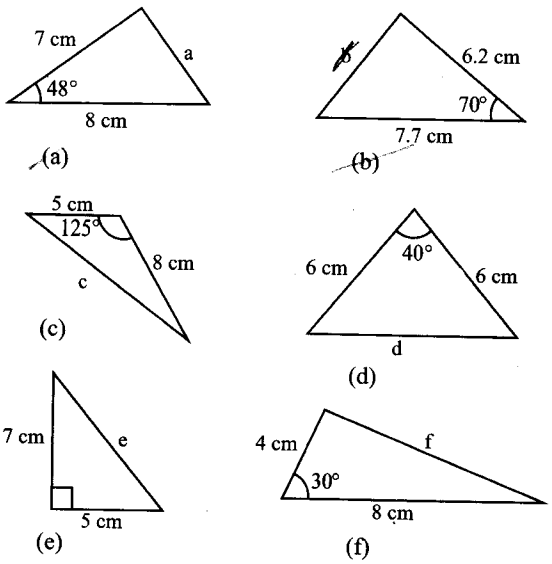



*Fig. 21-8*

**2.** Construct the following triangles. Two sides and the angles between them are given. Measure the third side.

(a) 7 cm, 48° , 9 cm

(b) 8.8 cm, 75° , 6.5 cm

(c) 5 cm, 110° , 6 cm

(d) 7.8 cm, 70° , 4.8 cm

(e) 9 cm, 40° , 6 cm

(f) 8.5 cm, 45° , 8.5 cm

**3.** Construct the given triangles.

(a) Δ ABC; AB = 7 cm, BC = 4.5 cm, ∠B = 105° . Measure AC.

(b) Δ PQR; PQ = 5.5 cm, QR = 4 cm, ∠R = 85° . Measure PR.

(c) Δ XYZ; XY = 5 cm, YZ = 6 cm, ∠X = 70° . Measure XZ.

(d) Δ JKL; JK = 5.5 cm, KL = 3.5 cm, ∠L = 130° . Measure JL.

(e) Δ STU; ST = 5 cm, TU = 6 cm, ∠T = 65°. Measure SU.

(f) Δ XYZ; XY = 6.5 cm, ZY = 7.5 cm, ∠X = 35°. Measure ∠Y.

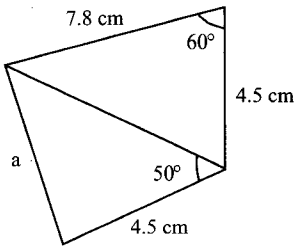
**4.** (a) Construct Δ ABC such that AB = 4.5 cm, BC = 3.5 cm and ∠A = 40°. Measure ∠C. Is there more than one possible triangle?

(b) Construct Δ ABC such that ∠AB = 4.5 cm, ∠A = 40° and (i) BC = 4.8 cm (ii) BC = 2.5 cm

Measure c. What do you notice?

**5.** Construct the diagrams in Fig.21-9 and measure the sides marked with letters.

(a)




(b)

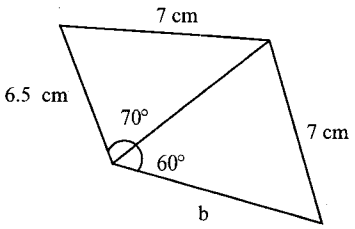



*Fig.21-9*

You should have noticed that:

When two sides and the angle between them (i.e. two sides and the included angle) are given, we are always able to construct a unique triangle (as in Question 2). But if the angle is not the included angle, we do not necessarily get a unique triangle (Question 4).

## Construction of a triangle, given one side and two angles

Construct Δ PQR such that PQ = 5 cm, ∠P = 35°, ∠R = 65°.

*Procedure:*

(a) Make a rough sketch of the triangle (Fig.21-10). Note that ∠Q = 80°. Why?

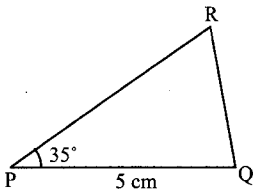



*Fig.21-10*

(b) Make an accurate construction as follows.

(i) Draw a line and mark a point P on it.

(ii) On the line, mark off a point Q, 5 cm from P, using a pair of compasses.

(iii) At point P, draw a line at an angle of 35°.

(iv) At point Q, draw a line at an angle of 80°.

Where the two lines meet is the point R.

Check that ∠R = 65°. PQR is the required triangle (Fig.21-11).

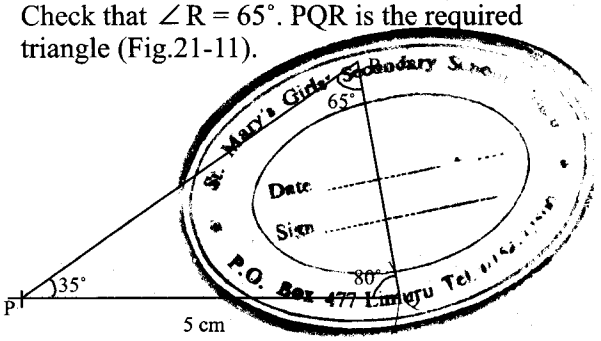



*Fig.21-11*

Note that given two angles and a side, we need to find the third angle so that the given side is between two angles. It is only then that we shall be able to construct the triangle.

### Exercise 21.3

**1.** Construct the triangles sketched in Fig.21-12 and measure the sides marked with letters.

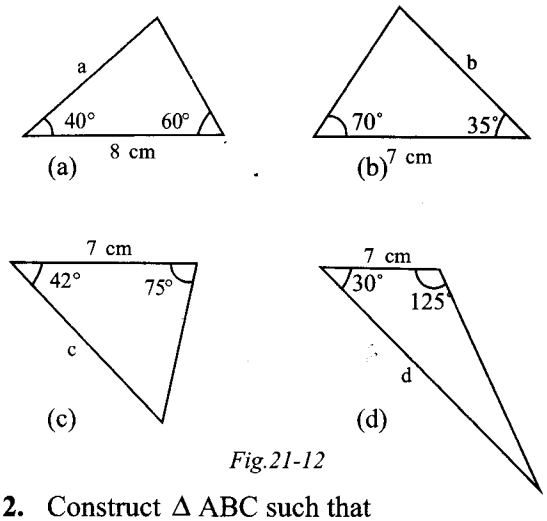



*Fig.21-12*

**2.** Construct Δ ABC such that

(a) $\angle A = 30°$, AB = 5.5 cm, $\angle B = 70°$. Measure AC.

(b) $\angle A = 25°$, $\angle B = 65°$, AC = 6.5 cm. Measure AB.

(c) $\angle A = 45°$, $\angle C = 40°$, AB = 7.8 cm. Measure AC.

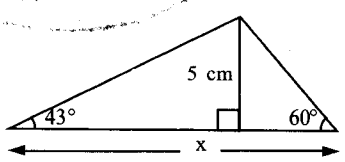
(d) AC = 8 cm, $\angle B = 55°$, $\angle C = 65°$. Measure BC.

(e) BC = 6.8 cm, $\angle B = 65°$, $\angle A = 45°$. Measure AB.

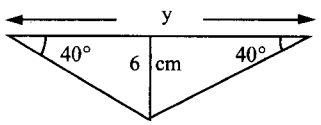
(f) BC = 6 cm, $\angle A = 35°$, $\angle B = 115°$. Measure AB.

**3.** Construct each of the shapes in Fig 21-13 and find the lengths marked with letters.

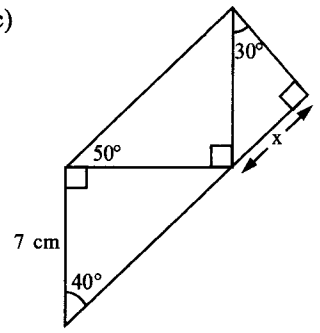
(a)




(b)




(c)




(d)

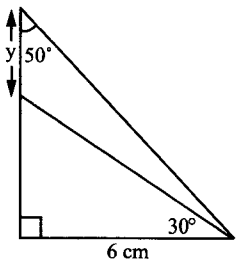
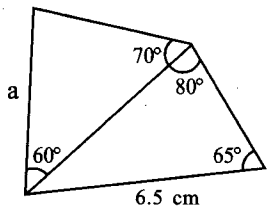


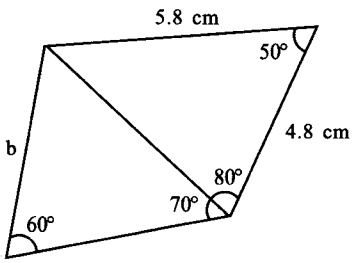

*Fig.21-13*

**4.** Construct the shapes in Fig.21-14 and measure the sides marked with letters.

(a)




(b)




(c)

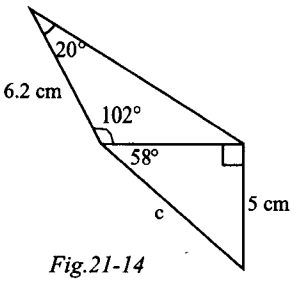



*Fig.21-14*

5. On the same diagram, construct triangles PQS and PRS from the given measurements.

(a) PQ = 5 cm, PR = 6 cm, $\angle Q = 40°$, QS = 7 cm, RS = 3 cm, Measure QR.

(b) PQ = 5 cm, QR = 9 cm, RS = 3 cm, SP = 3.8 cm, $\angle QPS = 100°$. Measure angle Q.

(c) PS = 5 cm, $\angle RPS = 35°$, $\angle PSQ = 47°$, $\angle PRS = 65°$, $\angle PQS = 54°$. Measure QR.

What is the name of the plane figure PQRS in (a), (b) and (c)?

## Construction of perpendicular lines

When two lines meet at right angles, they are said to be **perpendicular** to each other. For example, the adjacent edges of a text book are perpendicular to each other.

### Constructing a perpendicular to a line from a given point on the line

Construct a line through point O perpendicular to line AB (Fig.21-15).

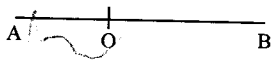



*Fig.21-15*

*Procedure:*

(a) Draw line AB and mark the point O, anywhere between A and B.

(b) With centre O and a convenient radius draw arcs to cut AB at P and Q (Fig.21-16).

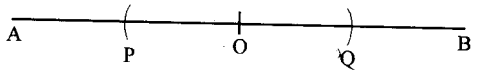



*Fig.21-16*

(c) With centre P and radius greater than PO draw an arc on one side of AB.

Using the same radius and centre Q draw an arc to cut the first arc at point R. Join RO. (Fig. 21-17) Check that $\angle AOR$ and $\angle BOR$ are 90° each.

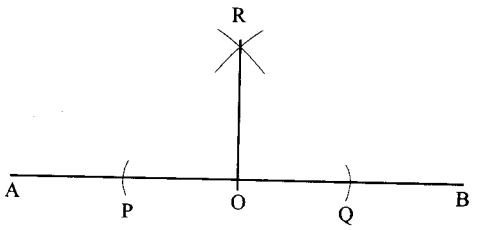



*Fig.21-17*

### Exercise 21.4

Use a ruler and a pair of compasses only in this exercise.

1. Given that AB = 8 cm, and that O is a point on AB such that AO = 3 cm, construct a line CO = 4 cm which is perpendicular to AB. Measure AC and BC.
2. Draw line PQ = 8 cm. Draw PR = 5 cm such that PR is perpendicular to PQ. Measure RQ.
3. Draw line PQ = 7 cm.
   (a) Draw QR = 3 cm such that QR is perpendicular to PQ.
   (b) Draw PS = 8 cm such that PS is perpendicular to PQ.
   Join and measure RS. What is the name of figure PQRS?
4. Given AD = 6 cm, construct an equilateral triangle ABC of side 6 cm such that AD is perpendicular to AB. Measure CD.
5. Make an accurate drawing of the shape shown in Fig.21-18 and measure $\angle ADE$

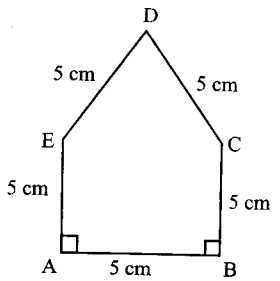



*Fig.21-18*

6. Construct

(a) a square of side 4 cm

(b) a rectangle of sides 4 cm by 7 cm.

## Constructing a perpendicular to a line from an external point using ruler and a pair of compasses

Construct a perpendicular from point M to line PQ (Fig.21-19).

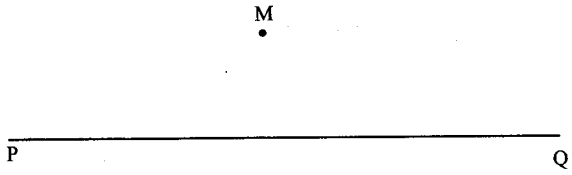



*Fig.21-19*

*Procedure:*

(i) With centre M and a suitable radius on a pair of compasses, draw arcs to cut line PQ at two points R and S (see Fig.21-20).

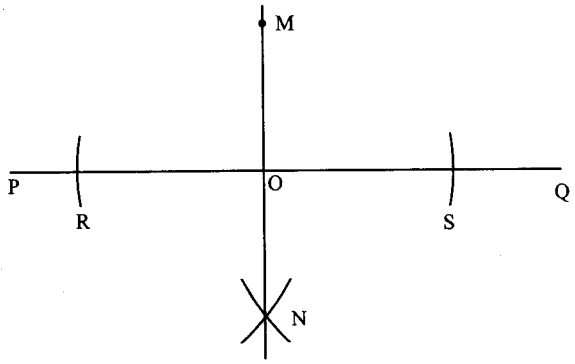



*Fig.21-20*

(b) With centre R and a convenient radius, draw an arc on the other side of PQ away from M.

(c) With centre S and the same radius, draw an arc to cut the first arc at N. Join MN (Fig.21-20).

Check that $\angle MOP$ and $\angle MOQ$ are each $90°$.

Do you think the construction would have been possible if the arcs were on the same side of PQ as M (but not through M)?

Note that the length MO is called the **perpendicular distance** of point M from the line PQ. It is also the **shortest distance** of point M from line PQ.

## Constructing a perpendicular to a line from an external point using a ruler and a set square

In Fig.21-20, the perpendicular from point M to line PQ could have been drawn using a ruler and a set square as follows.

(a) Place a ruler along the given line PQ (Fig.21-21).

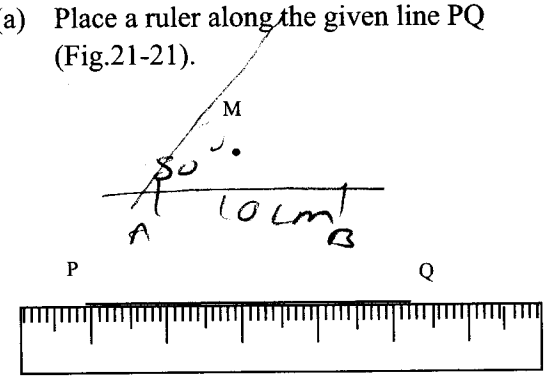



*Fig.21-21*

(b) Place a set square so that it rests against the ruler in any position $S_1$ (Fig.21-22). Hold the ruler firmly and slide the set square along the ruler until the edge reaches M (position $S_2$).

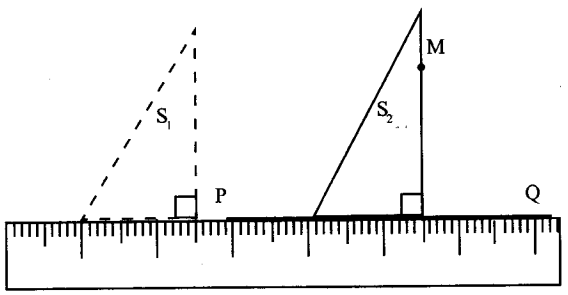



*Fig.21-22*

(iii) Hold the set square firmly in position $S_2$, remove the ruler and draw a line through M to cut line PQ, at a point O.

This is the required perpendicular (Fig.21-23).

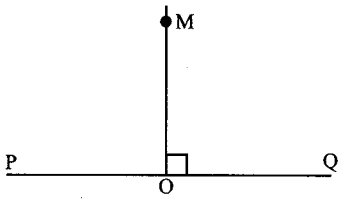
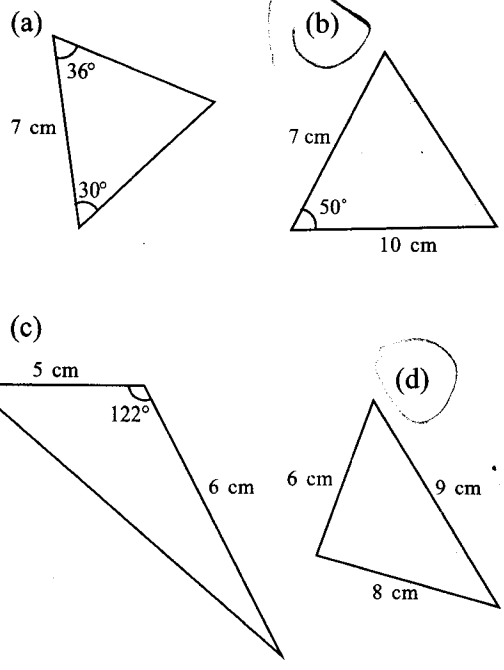



*Fig.21-23*

## Exercise 21.5

1. Draw any line AB. Mark a point P on one side of AB. Draw a perpendicular from P to AB using ruler and compasses. Confirm your working using a ruler and a set square.

2. Draw each of the triangles in Fig.21-24 accurately. From one of the vertices construct a perpendicular to the opposite side. Measure the perpendicular and use it to find the area of the triangle.




(e)

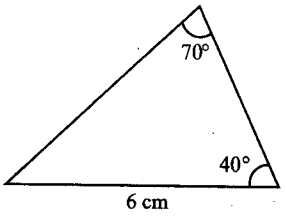



*Fig.21-24*

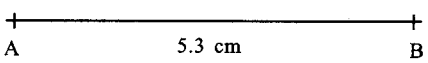
## Bisecting a given line segment

Bisect a line segment AB whose length is 5.3 cm.

*Procedure:*

(a) Draw a line and on it, mark points A and B, 5.3 cm apart (Fig.21-25(a)).

(a)




(b)

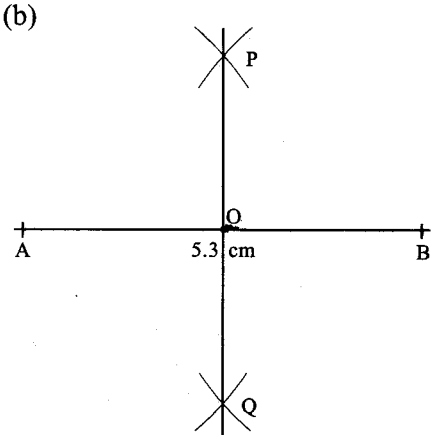



*Fig.21-25*

(b) With centre A and a radius greater than half the length of AB, mark arcs on either side of AB. With centre B, and the same radius, mark arcs which cut the first arcs at points P and Q.

(c) Join PQ (Fig 21.25(b)).

Check that AO = OB and $\angle POB = 90°$.

Line PO (or PQ) is called the **perpendicular bisector** of AB or the **mediator** of AB.
Any point on the mediator is **equidistant** (i.e. at equal distances) from points A and B. Check this!

**Exercise 21.6**

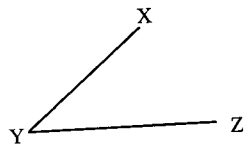
1. Draw line AB = 7.5 cm. Find its mid-point by bisection.
2. (a) Draw any scalene Δ ABC. Construct the perpendicular bisectors of all the sides. What do you notice?
   (b) Draw any equilateral triangle Δ ABC. Construct the perpendicular bisectors of all the sides. What do you notice?
   (c) In both (a) and (b) above, measure the distances from the vertices to the point where the perpendicular bisectors meet. What do you notice?
3. Draw any circle centre O. Draw two chords PQ and RS which are not diameters. Construct perpendicular bisectors of the chords. Where do they meet?
4. Draw any equilateral triangle Δ ABC.
   (a) Construct perpendicular bisectors of AB and AC.
   (b) Construct perpendiculars from A to BC and B to AC.
   What do you notice?
5. On a clean page mark points A, B and C such that they are not on a straight line. Find a point O such that AO = BO = CO

You should have noticed that:

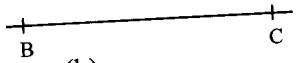
(i) the perpendicular bisector of a chord passes through the centre of the circle.
(ii) the perpendicular bisectors of the sides of an equilateral triangle are also the **altitudes** of the triangle.

## Copying an angle using a ruler and compasses only

Construct $\angle ABC$ on line BC equal to $\angle XYZ$, Fig.21-26.




(a)



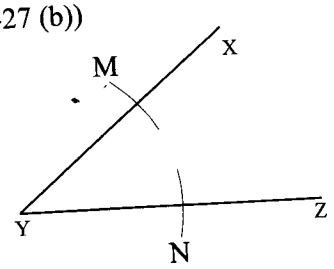

(b)

*Fig.21-26*

*Procedure*

(a) With Y as centre and any radius, draw an arc to cut XY and YZ at M and N respectively (Fig.21-27 (a)).
(b) With YN as radius and B as centre, draw an arc to cut BC at N′ (Fig.21-27 (b)).
(c) With MN as radius and N′ as centre draw an arc to cut the first arc in (b) above at M′.
(d) Join B to M′. The required angle is ABC (Fig 21-27 (b))

(a)




(b)

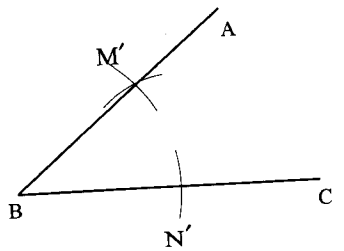



*Fig.21-27*

**Exercise 21.8**

In this exercise, use a ruler and a pair of compasses only.

1. Draw an angle ABC and a straight line DE. Construct a line DF such that $\angle EDF = \angle BAC$.
2. Draw an angle PQR. Draw an isosceles triangle KLM such that $\angle KLM = \angle PQR$.
3. Draw an angle PQR. Draw a right angled $\Delta$ ABC such that $\angle ABC = \angle PQR$.
4. Draw a triangle ABC. Draw triangle ACD such that $\angle DAC = \angle BCA$ and $\angle ACD = \angle BAC$. What type of shape is figure ABCD?
5. Draw a triangle PQR and take a point X inside the triangle. Construct a point Y on QR such that $\angle XYR = \angle PQR$.

## Bisecting an angle

Draw any angle ABC and bisect it using a ruler and pair of compasses only.

*Procedure:*

(a) With centre B and any radius, draw arcs to cut AB at P and BC and Q.

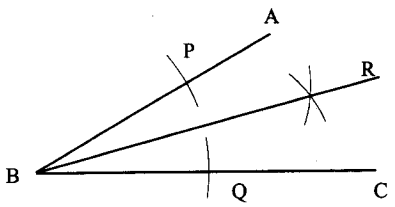



*Fig.21-28*

(b) With centre P and any suitable radius, draw an arc between the lines AB and BC.

(c) With centre Q and same radius used in (b) above, draw an arc to cut the first arc at R.

(d) Join BR.

Measure $\angle ABC$ and $\angle RBC$. You should find that $\angle ABC = 2\angle RBC$.

BR is called the **angle bisector** of $\angle ABC$.

**Exercise 21.8**

In this exercise, use a ruler and pair of compasses only.

1. Draw any triangle ABC and bisect $\angle ABC$. Use a protractor to check your working.
2. Draw any equilateral triangle and bisect all the angles. What do you notice?
3. Draw any scalene triangle and bisect all the angles. What do you notice?
4. Draw an obtuse angle. Divide it into four equal angles.
5. Draw a circle of any radius and mark three points P, Q, R on the circumference.
   (a) Construct the bisector of $\angle PQR$ and the perpendicular bisector of PR. Where do they meet?
   (b) Construct the bisector of $\angle PRQ$ and the perpendicular bisector of PQ. Where do they meet?

You should have observed that:

> Angle bisectors of a triangle meet at a common point.

## Constructing angles of 30°, 45°, 60°, 90°

(a) To construct an angle of 90°, draw a perpendicular to a line from a given point on the line as shown in Fig. 21-17.

(b) To construct an angle of 45°, first construct an angle of 90°, then bisect it.

(c) To construct an angle of 60°:
   (i) Draw a line AB.
   (ii) With centre A and any radius, draw an arc to cut AB at X (Fig.21-29).

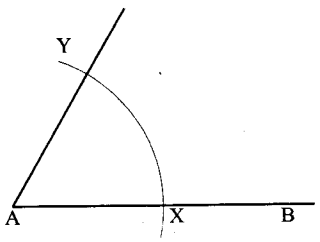



*Fig.21-29*

(iii) With centre X and the same radius used in (ii) above, draw an arc to cut the first arc at Y.

(iv) Join AY. $\angle YAB = 60°$. Measure $\angle YAB$.

(d) To construct, an angle of 30°, first construct an angle of 60°, then bisect it.

(e) To construct angles such as 15°, 75°, 105°, 120°, etc, we construct the relevant special angles above and either bisect or combine them.

**Exercise 21.9**

1. Construct the following angles.
   (a) 60° (b) 30° (c) 45°
   (d) 75° (e) 120° (f) 135°
   (g) $22\frac{1}{2}°$ (h) 15°
2. Construct a right angle and construct two lines dividing the right angle into three equal angles.
3. Construct $\Delta ABC$ such that AC = 10 cm, $\angle BAC = 30°$, $\angle ABC = 90°$. Construct a circle passing through the vertices. What can you say about AC?
4. Construct $\Delta ABC$ such that $\angle ABC = 67.5°$ BC = 8 cm, $\angle BCA = 75°$. Measure AC.
5. Construct isosceles $\Delta ABC$ such that AB = BC = 6.4 cm, $\angle ABC = 105°$. Measure AC.

## Constructing parallel lines

### (a) Using a ruler and a pair of compasses only

Given line AB, construct a line through P parallel to AB (Fig.21-30)

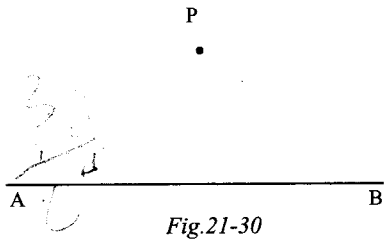



*Fig.21-30*

*Method 1*

(i) On line AB mark two points X and Y.

(ii) With centre P and radius XY, draw an arc.

(iii) With centre X and radius PY, draw an arc to intersect the first arc at Q (Fig.21-31).

(iv) Join PQ. PQ is the required line.

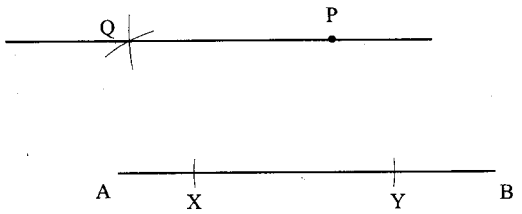



*Fig.21-31*

*Method 2*

(i) Take any point X on AB. Join PX.

(ii) With centre X and any radius, draw an arc to cut AB at M and PX at N.

(iii) With centre P and same radius as in (ii) draw an arc to cut PX at K.

(iv) With centre K and radius MN, draw an arc to cut the other arc at L.

(v) Join PL. This is the required line (Fig.21-32).

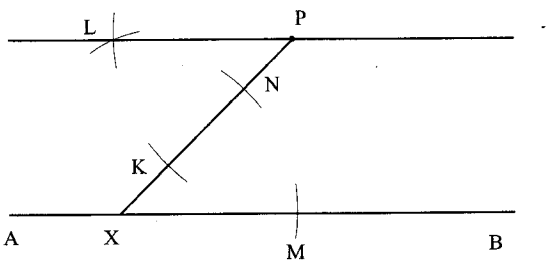



*Fig 21.32*

### (b) Using a ruler and a set square

(i) To draw a line parallel to line AB, passing through a given point P, place a set square along AB.

(ii) Place a ruler against the set square as shown in Fig.21-33.

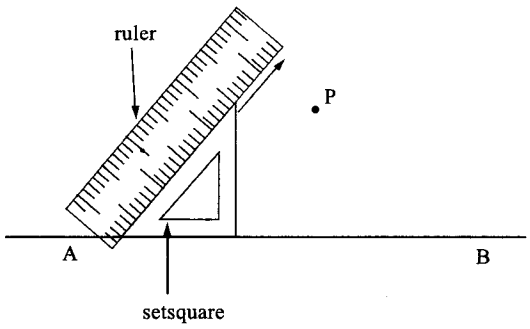



*Fig.21-33*

(iii) Hold the ruler firmly, then slide the set square in the direction shown by the arrow (Fig.21-33) until it reaches the position shown in Fig.21-34.

(iv) Hold the set square in position and draw a line through P.

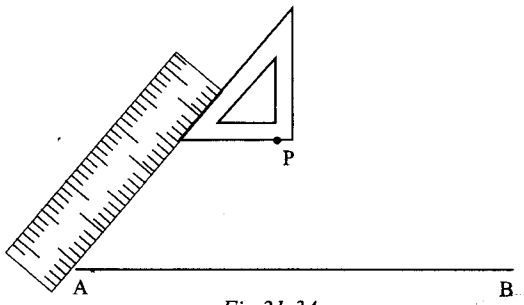



*Fig.21-34*

This will be the required line (Fig.21-35).

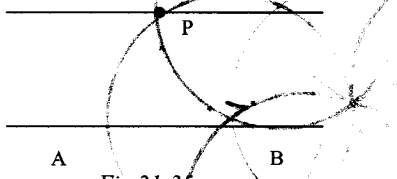



*Fig.21-35*

## Proportional division of a line segment

A line segment can be divided into a given number of equal parts or in a given ratio as shown below.

### Line divided into equal parts

Divide line segment AB in Fig.21-36 (a) into 5 equal parts.

*Procedure:*

(a) Through A, draw any line AL.

(b) Using a suitable radius on a pair of compasses, starting at A, mark off 5 equal lengths $AA_1$, $A_1A_2$, $A_2A_3$, $A_3A_4$, $A_4A_5$, along AL.

(c) Join $BA_5$.

(d) Using a ruler and set square, draw lines $A_1B_1$, $A_2B_2$, $A_3B_3$, $A_4B_4$ parallel to $A_5B$. (Fig.21-36).

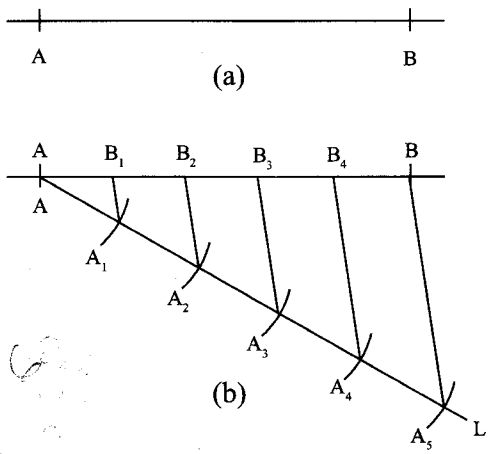



*Fig.21-36*

$B_1$, $B_2$, $B_3$, $B_4$ are the points that divide AB into the required 5 equal parts.

### Line divided in a given ratio

Draw line PQ = 5 cm and divide it in the ratio 1:2.

*Procedure:*

(a) Draw line PQ = 5 cm

(b) Draw line PR (3 cm or 6 cm, or 9 cm, etc. long) at an acute angle to PQ and divide it into 3 equal parts.

(c) Join RQ

(d) Lengths $PR_1$, and $R_1R$ will then be in ratio 1:2. Hence draw $R_1X$ parallel to RQ.

PX:XQ = 1:2 (Fig.21-37).

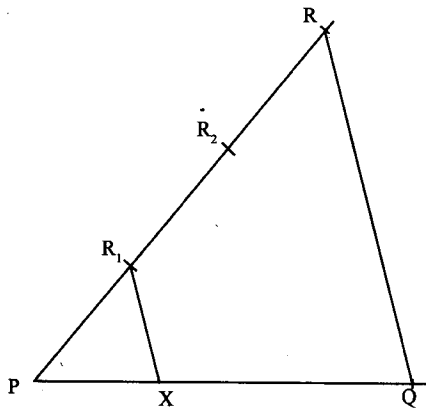



*Fig.21-37*

**Exercise 21.10**

1. Draw a straight line PQ. Mark a point X above the line. Construct a line through X parallel to PQ using a ruler and a setsquare only .
2. Draw a line XY. On it, mark a point M. Through point M, construct a perpendicular. On this perpendicular, mark off points A and B such that MA = AB = 3 cm.Through points A and B, construct lines parallel to the line XY. (Use a ruler and a pair of compasses only ).
3. Draw a line PQ, 6 cm long. Construct $\angle SPQ = 60^\circ$ with PS = 4 cm. Through points S and Q, construct lines parallel to PQ and PS respectively, to meet at a point R. Measure SR and QR, and all the remaining angles. Name the figure obtained.
4. Draw any $\Delta ABC$. Construct D such that AD = BC and CD = BA. What shape is the figure?
5. Draw a line AB = 8 cm long. Find a point X on AB such that AX : XB = 5 : 4. Measure AX.
6. Draw a line PQ = 10 cm long. Find a point D on line PQ such that PD : DQ = 2 : 7. Measure DQ.
7. Using a ruler and compasses only, draw $\Delta$ ABC such that AB = 8.4 cm, BC = 7.7 cm, AC = 10.5 cm. P and Q are points on AB and AC respectively such that AP : PB = AQ : QC = 3 : 4. Measure PQ and find the ratio

   (a) PQ : BC (b) AP : AB.
8. Construct $\Delta$ ABC such that AB = 8 cm, BC = 10 cm, CA = 9 cm. Find a point X on AB such that AX = XB. Through X draw line XY parallel to BC and meeting AC at Y. Measure XY. Find the ratio

   (a) XY : BC (b) AX : AB

## Constructing a square

Construct a square of side 3 cm.

*Procedure:*

(b) (i) Draw a line and mark off points A and B, 3 cm apart.
(ii) At points A and B, draw lines perpendicular to line AB.
(iii) With centre A and radius 3 cm, draw an arc on the perpendicular cutting it at point D.
(iv) With centre B and radius 3 cm, draw an arc to cut the perpendicular at C.
(v) Join CD (Fig.21-38 (b)).
ABCD is the required square.

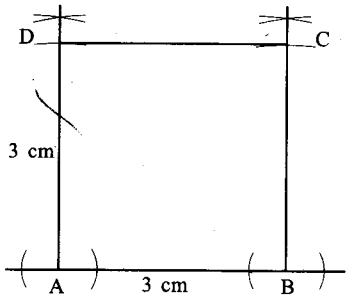



*Fig. 21-38*

## Construction of a rectangle

Construct a rectangle 5 cm long and 3 cm wide.

*Procedure:*

*Procedure:*

(a) Draw a line and mark off points P and Q, 5 cm apart.

(a) At point P, construct a line perpendicular to PQ.

(c) With P as the centre, and radius 3 cm, draw an arc to cut the perpendicular at S.

(d) At point Q, construct a line perpendicular to line PQ.

(e) With centre Q, and radius 3 cm, draw an arc to cut the perpendicular at R

(f) Join SR. (Fig.21-39(b)).

PQRS is the required rectangle.

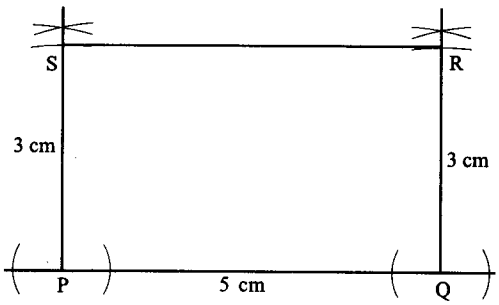



*Fig.21-39*

## Construction of a rhombus

Construct a rhombus whose diagonals are 4.5 cm and 3 cm.

*Procedure:*

(a) Draw a rough sketch of the rhombus (Fig.21-40(a))

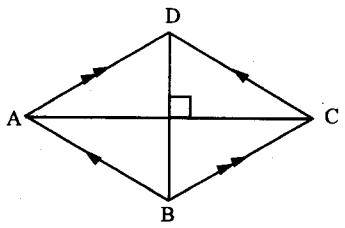



*Fig.21-40 (a)*

(b) (i) Draw a line and mark off points A and C, 4.5 cm apart.

(ii) Construct a perpendicular bisector of line segment AC to cut the line at point O.

(iii) With centre O and radius 1.5 cm, draw arcs on either side of the perpendicular bisector at points B and D.

(iv) Join AB, BC, CD and DA (Fig.21-40 (b)).

ABCD is the required rhombus.

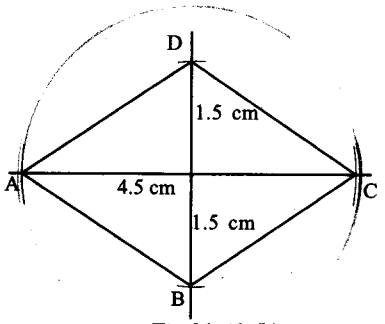



*Fig.21-40 (b)*

## Constructing a parallelogram

To construct a parallelogram, two adjacent sides and an included angle have to be known.

Construct a parallelogram whose adjacent sides are 3.5 cm and 4.5 cm and the included angle is 45°.

*Procedure:*

(a) Draw a rough sketch of the parallelogram (Fig.21-41(a)).

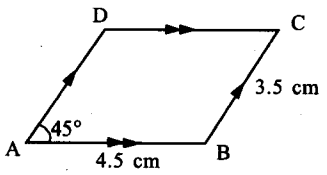



*Fig.21-41 (a)*

(b) (i) Draw a line and mark off points A and B, 4.5 cm apart.
(ii) Construct a line at $A_1$, making an angle of 45° to AB.
(iii) With centre A and radius 3.5 cm, draw an arc to cut the line in (ii) at D.
(iv) With centre at B and radius 3.5 cm, draw an arc above line AB.
(v) With centre D and radius 4.5 cm, draw an arc which cuts the arc in (iv) at point C.
(vi) Join BC and DC (Fig.21-41(b)) . ABCD is the required parallelogram.

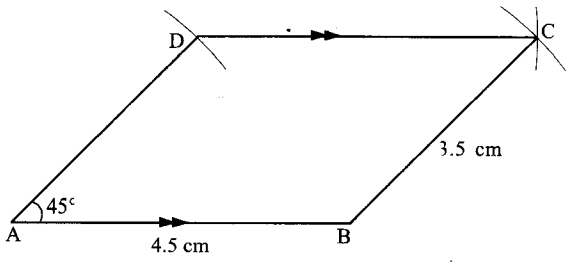



*Fig. 21-41(b)*

## Constructing a trapezium

Note that a trapezium has two sides parallel to each other.
Construct a trapezium with sides AB = 6.0 cm, BC = 4.0 cm, CD = 3.5 cm, DA = 3.0 cm and AB // DC. Measure BD.

*Procedure:*
(a) Draw a rough sketch of the trapezium (Fig.21-42(a)).

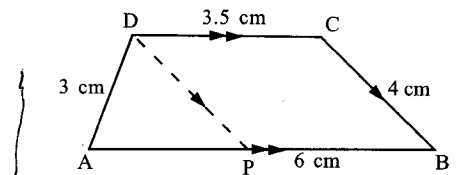



*Fig. 21-42 (a)*

(b)
(i) On the sketch draw DP parallel to CB to meet AB at P.
(ii) Draw a line and mark off points A and B, 6 cm apart.
(iii) With centre B and radius 3.5 cm, draw an arc cutting AB at P.
(iv) With centre P and radius 4 cm, draw an arc.
(v) With centre A and radius 3 cm, draw an arc cutting the arc drawn in (iv) at point D.
(vi) With centre D and radius 3.5 cm, draw an arc.
(vii) With centre B and radius 4 cm, draw an arc cutting the arc drawn in (vi) at point C. Join BC, CD and AD (Fig.21-42 (b)). ABCD is the required trapezium. BD = 6.9 cm.

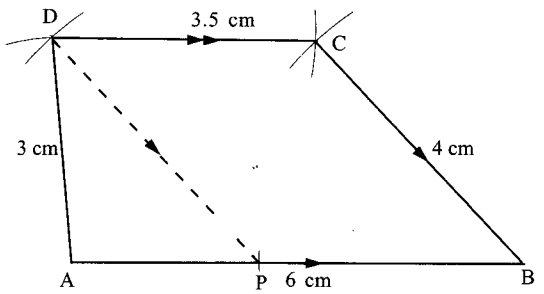



*Fig.21-42 (b)*

### Exercise 21.11

In this exercise, use a ruler and a pair of compasses only.

1. Construct a square with sides.
   (a) 4.5 cm. (b) 6 cm.
   Measure the diagonals.
2. Construct a rectangle with sides
   (a) 4 cm and 7.5 cm
   (b) 5.5 cm and 8.5 cm
   Measure the diagonals in each case.
3. Construct a rhombus whose diagonals are
   (a) 8 cm and 4 cm. (b) 5 cm and 7 cm.
   Measure the sides in each case.

4. Construct a parallelogram with

   (a) adjacent sides 6.5 cm and 3.8 cm and the included angle 52.5°. Measure the longer diagonal.

   (b) two sides 7.5 cm and 4.8 cm and one diagonal 10.8 cm. Measure the smaller angle in the parallelogram.

5. Construct a trapezium ABCD such that AB is parallel to DC, AB = 5 cm, BC = 5.5 cm, AD = 6.5 cm and DC = 10 cm. Measure AC and ∠ACD.

## Constructing regular polygons

### (a) Constructing a regular polygon from the centre of a circle

A regular polygon has equal sides. If the vertices are joined to the centre of the polygon, the number of triangles formed are equal to the number of sides of the polygon. The number of angles at the centre will also be equal to the number of sides of the polygon and are of equal size.

Each angle at the centre is $\frac{360°}{n}$.

**Draw a regular pentagon using a circle of radius 2 cm.**

*Procedure:*

(a) With centre O, draw a circle radius 2 cm. Draw a radius OA (Fig. 21-43).

(b) From centre O, mark an angle of 72° as shown i.e. ∠BOA = 72°, where A and B are on the circumference of the circle.

(Angle at the centre = $\frac{360°}{5}$ = 72°)

(c) With centre B and radius AB, draw an arc cutting the circle at point C.

(d) With centre C and the same radius, draw an arc cutting the circle at D.

(e) With centre D and the same radius, draw an arc cutting the circle at E.

(f) Join AB, BC, CD, DE, EA (Fig.21-43).

ABCDE is the required regular pentagon.

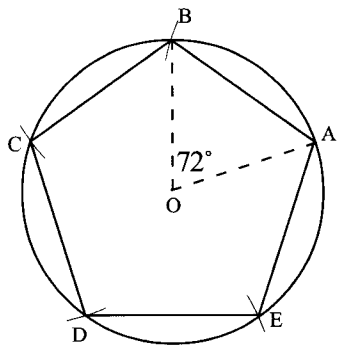



*Fig.21-43*

### (b) Construction of a regular polygon on a given line

A regular polygon has all its interior angles equal. Remember that the sum of interior angles in an n-sided polygon is (2n – 4) right angles.

$\therefore$ One interior angle = $\frac{(2n-4) \text{ right angles}}{n}$

**Construct a regular pentagon of sides 3 cm.**

*Procedure:*

(a) Draw a line PQ = 3 cm long.

(b) At Q, construct an angle of 108°.

One interior angle = $\frac{(10-4)\,90°}{5}$

$= 108°$

(c) With centre Q and radius 3 cm, make an arc on the line drawn in (b) to cut it at point R.

(d) At R make an angle of 108°. With the same radius drawn in (c) make an arc to cut the line drawn at S.

(e) Repeat steps (b) and (c) at S to get point T.

(f) Join PT (Fig.21-44).

PQRST is the required pentagon.

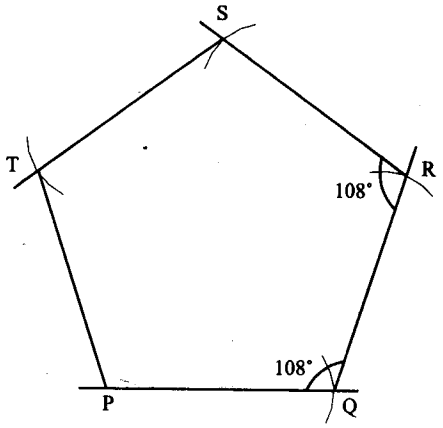



*Fig. 11-44*

**Recall that:**

A polygon with n sides has n exterior angles. If the polygon is regular, then the exterior angles are equal, each being $\frac{360°}{n}$.

Thus, the following is an alternative procedure for constructing a regular polygon on a given line.

*Procedure*

(a) Draw a line of the specified length.
(b) At one end of the line, make an exterior angle of $\frac{360°}{n}$ (Fig. 11-45).
(c) On the new line, mark off a length equal to the length of the side.
(d) At the end of the line drawn in (c), draw an exterior angle, as in (b).
(e) Continue on each new line till the required polygon is formed.

Now repeat the construction of a regular pentagon using this procedure.

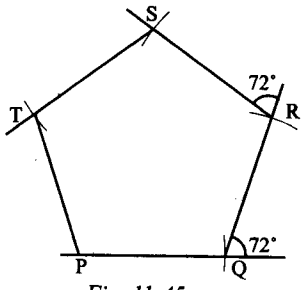



*Fig. 11-45*

## Constructing an irregular polygon

In an irregular polygon, the sides are not equal. Thus, to construct one of n sides, the sizes of at least (n – 2) consecutive angles and lengths of (n – 1) sides must be known.
The procedure then is the same as that of constructing a regular polygon, only that the angle and length to be marked off at each vertex keep changing.

### Exercise 21.12

1. By first drawing a circle, make an accurate drawing of a
   (a) regular quadrilateral,
   (b) regular hexagon,
   (c) regular octagon.
2. Construct a regular octagon of side 3.5 cm. Measure its interior angles.
3. Construct a regular decagon of side 3 cm. Measure its interior angles.
   construct a regular polygon with 12 sides (duodecagon).
5. Using a ruler and a pair of compasses only, Construct a regular octagon starting with a circle of radius 4.5 cm. Measure one side of the octagon.
6. Construct a pentagon ABCDE such that AB = 3.4 cm, BC = 3.8 cm, $\angle ABC = 110°$, CD = 5.5 cm, $\angle BCD = 125°$, DE = 3.2 cm, $\angle CDE = 80°$. Measure AE and $\angle AED$.