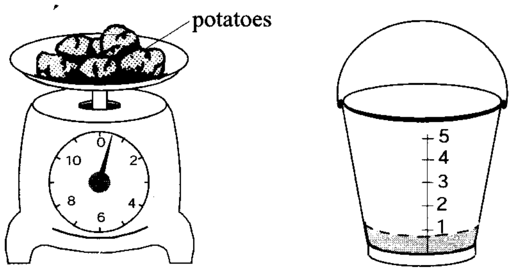
6 FRACTIONS

What are fractions?

In many practical situations, parts of whole numbers are required. Parts of whole numbers are called **fractions**. Sharing, measuring lengths, measuring volumes, measuring mass, etc, give rise to fractions of whole numbers.

Fig. 6-1 shows a scale balance and a graduated transparent bucket.



(a) (b)

Fig. 6-1

The mass of the potatoes is between 0 and 1 kg ($\frac{1}{2}$ kg).

The bucket contains milk between 0 and 1 litre ($\frac{3}{4}$ litre).

Fig. 6-2(a) shows a cane. Fig. 6-2(b) shows the same cane cut into 5 equal parts.

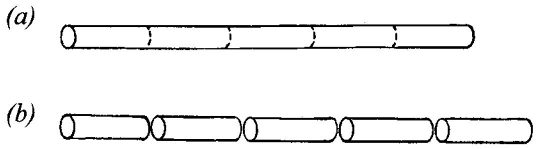


Fig. 6-2

Each part in (b) is a fraction of the whole cane. It represents one out of five parts, written as $\frac{1}{5}$ and read as 'one over five' or simply 'one fifth'. 5 shows the number of parts into which the cane is divided equally, while 1 shows the number of parts taken.

1 part represents $\frac{1}{5}$ of the whole
2 parts represent $\frac{2}{5}$ of the whole
3 parts represent $\frac{3}{5}$ of the whole
4 parts represent $\frac{4}{5}$ of the whole
5 parts represent $\frac{5}{5}$ of the whole

In a fraction, the top number is called the **numerator** and the bottom number is the **denominator**.

Example 6.1

Write the following in words.

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{5}{8}$

Solution

(a) 'one over two' or 'a half'.
(b) 'two over three' or 'two thirds'.
(c) 'three over four' or 'three quarters'.
(d) 'five over eight' or 'five eighths'.

Exercise 6.1

1. The diagrams in Fig. 6-3 are divided into equal portions. What fraction of the whole do(es) the shaded part(s) in each diagram represent?

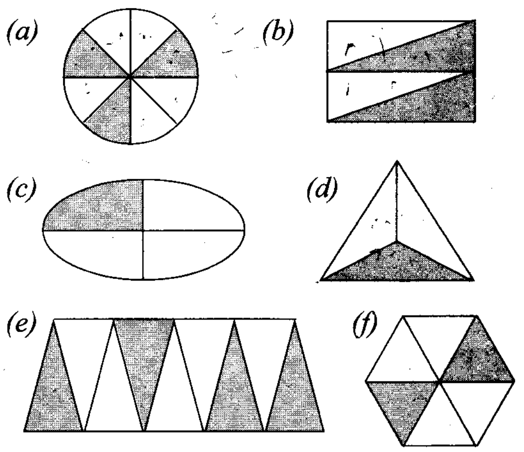


Fig. 6-3

2. Write the following as fractions.
   (a) one half (b) seven eigths
   (c) nine twentieths (d) eleven twelfths
   (e) seven tenths (f) two nineths

3. Write the following fractions in words.
   (a) $\frac{8}{9}$ (b) $\frac{5}{6}$ (c) $\frac{2}{3}$
   (d) $\frac{6}{11}$ (e) $\frac{7}{18}$ (f) $\frac{4}{15}$

4. How many
   (a) sixteenths make a whole?
   (b) twelfths make a whole?
   (c) eighths make a whole?

Types of fractions

(i) A fraction whose numerator is smaller than its denominator is called a **proper fraction.**

e.g. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{5}{8}, \frac{6}{8}, \frac{6}{7}$ are proper fractions

(ii) A fraction whose numerator is greater than its denominator is called an **improper fraction**

e.g. $\frac{3}{2}, \frac{5}{2}, \frac{4}{3}, \frac{7}{3}, \frac{8}{5}$ are improper fractions

(iii) A number made of a whole number part and a proper fraction part is called a **mixed number.**

e.g. $1\frac{1}{2}, 1\frac{1}{3}, 2\frac{1}{4}, 3\frac{1}{3}, 2\frac{3}{5}$ are mixed numbers

Note: Proper and improper fractions are collectively called **common fractions** or **vulgar fractions.**

The number line in Fig. 6-4 shows whole numbers from 0 to 4. Each unit interval is divided into 8 equal parts. Point A can be assigned the number $\frac{4}{8}$ or $\frac{1}{2}$. What number would you assign to each of the points marked with letters B to G?

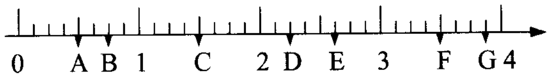


Fig. 6-4

Examples 6.2 and 6.3 illustrate how to convert improper fractions to mixed numbers and vice versa.

Example 6.2

Write the improper fraction $\frac{15}{6}$ as a mixed number.

Solution

$\frac{15}{6}$ *means* $15 \div 6 = 2$ *rem* 3

$\therefore \frac{15}{6} = 2\frac{3}{6}$ *(since the remainder is also divided by 6)*

Example 6.3

Express the mixed number $3\frac{2}{5}$ as an improper fraction.

Solution

$3\frac{2}{5}$ *means 3 wholes and two fifths*

$\therefore 3\frac{2}{5} = \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{2}{5}$

= *five fifths + five fifths + five fifths + two fifths*

= *17 fifths* = $\frac{17}{5}$

This may be done simply as follows:

$$3\frac{2}{5} = \frac{3 \times 5 + 2}{5} = \frac{15 + 2}{5} = \frac{17}{5}$$

Point of interest

Farey sequence of fractions is a list of all proper fractions with denominator less than or equal to a given number. The given number is said to be the order of the sequence.

This sequence is named after John **Farey** (1766–1826), an English civil engineer and mathematician.

Example 6.4

Write all the possible proper fractions whose denominators are less than or equal to 3.

Solution

The fractions are $\frac{0}{1}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{1}{1}$.

This is the Farey sequence of order 3.

Note: You can make a Farey sequence of any order.

Exercise 6.2

1. Copy Table 6-1 and indicate the type of fraction that each of the numbers represents by ticking in the appropriate column.

Number	Proper fraction	Improper fraction	Mixed number
$\frac{1}{2}$			
$\frac{2}{3}$			
$\frac{27}{8}$			
$1\frac{1}{5}$			
$\frac{15}{9}$			
$\frac{11}{16}$			
$\frac{9}{8}$			
$2\frac{3}{8}$			

Table 6-1

2. Convert the following fractions into mixed numbers.

   $\frac{16}{3}, \frac{12}{5}, \frac{6}{5}, \frac{9}{7}, \frac{11}{4}, \frac{31}{6}, \frac{43}{8}, \frac{52}{9}, \frac{17}{12}$.

3. Convert the following mixed numbers into improper fractions.

   $1\frac{1}{2}, 12\frac{1}{3}, 16\frac{1}{2}, 6\frac{3}{5}, 7\frac{3}{7}, 15\frac{1}{3}, 17\frac{5}{8}$.

4. Write the shaded portions in Fig. 6.5 as improper fractions.

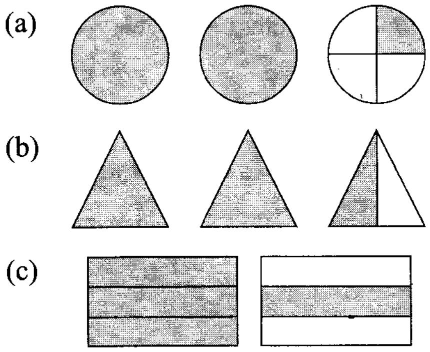


Fig. 6-5

5. Copy and complete Table 6-2 for Farey sequences of the indicated orders.

Order	*Farey sequence*	*Number of terms*
2		
3	$\frac{0}{1}, \frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{1}$	5
4		
5		

Table 6-2

Comparing fractions

Equivalent fractions

Fractions which represent the same point on the number line are called **equivalent fractions.** Fig. 6-6 shows the same line divided into two, four and eight equal parts. Point A can be assigned the value $\frac{1}{2}$, $\frac{2}{4}$ or $\frac{4}{8}$. These are equivalent fractions, i.e. fractions with the same value.

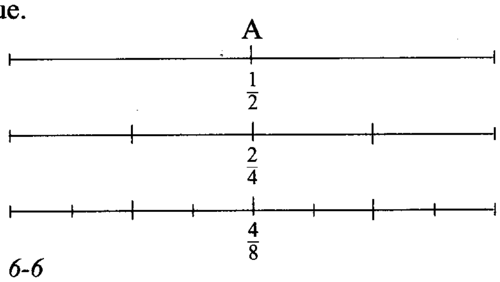


Fig. 6-6

Fig 6-7 is an alternative way of representing equivalent fractions. All the shaded portions represent equal fractions of the same circle.

Thus $\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{3}{6}$.

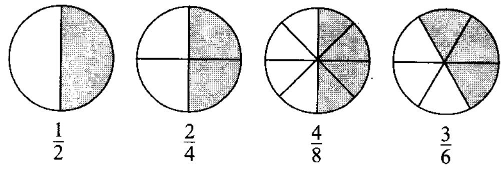


Fig. 6-7

The following are simple examples of how to obtain fractions equivalent to a given one.

$\frac{1}{2} \underset{\times 2}{\overset{\times 2}{=}} \frac{2}{4}$ Numerator is multiplied by 2
Denominator is multiplied by 2

$\frac{1}{2} \underset{\times 4}{\overset{\times 4}{=}} \frac{4}{8}$ Here the multiplier is 4

$\frac{1}{2} \underset{\times 3}{\overset{\times 3}{=}} \frac{3}{6}$ Here the multiplier is 3

$\frac{2}{4}$, $\frac{4}{8}$ and $\frac{3}{6}$ are obtained from $\frac{1}{2}$. Both the numerator and the denominator are multiplied by the same number. This common multiplier does not alter the value of the fraction.

Note that in some cases, we get fractions equivalent to a given one by dividing both the numerator and denominator by the same number.

Example 6.5

Complete the following to make equivalent fractions.

(a) $\frac{2}{3} = \frac{\square}{6}$ *(b)* $\frac{4}{7} = \frac{12}{\square}$

(c) $\frac{2}{3} = \frac{\square}{18} = \frac{20}{\square}$

Solution

(a) $\frac{2}{3} \underset{\times 2}{\overset{\times 2}{=}} \frac{\boxed{?}}{6} = \frac{\boxed{4}}{6}$

(b) $\frac{4}{7} \underset{\times 3}{\overset{\times 3}{=}} \frac{12}{\boxed{?}} = \frac{4 \times 3}{7 \times 3} = \frac{\boxed{12}}{21}$

(c) $\frac{2}{3} \underset{\times 6}{\overset{\times 6}{=}} \frac{\boxed{12}}{18}$, $\frac{2}{3} \underset{\times 10}{\overset{\times 10}{=}} \frac{20}{\boxed{30}}$ *(Since no whole number can multiply 12 to give 20, we use the first fraction again)*

Simplifying fractions

A fraction is in its simplest form if its numerator and denominator have no common factor, i.e. they are co-prime, e.g. $\frac{1}{2}$ is the simplest form of the equivalent fractions $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$, $\frac{6}{12}$, etc.

Example 6.6

Write the following fractions in their simplest forms.

(a) $\frac{8}{12}$ *(b)* $\frac{45}{30}$ *(c)* $7\frac{9}{12}$

Solution

(a) $\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$

(b) $\frac{45}{30} = \frac{45 \div 15}{30 \div 15} = \frac{3}{2}$

(c) $7\frac{9}{12} = 7 + \frac{9 \div 3}{12 \div 3} = 7\frac{3}{4}$

Exercise 6.3

1. Use the method of Example 6.5 to show that the following pairs of fractions are equivalent.
$\frac{10}{12}, \frac{30}{36}$; $\frac{2}{8}, \frac{1}{4}$; $\frac{5}{6}, \frac{35}{42}$; $\frac{4}{7}, \frac{12}{21}$; $\frac{3}{4}, \frac{6}{12}$.

2. Write the following in their simplest forms.
$\frac{1}{2}$, $\frac{4}{6}$, $\frac{14}{21}$, $\frac{8}{12}$, $\frac{9}{15}$, $\frac{8}{10}$, $\frac{6}{18}$, $\frac{70}{75}$, $\frac{26}{65}$, $\frac{22}{33}$, $\frac{6}{10}$.

3. Copy and complete the following to make equivalent fractions.
(a) $\frac{3}{7} = \frac{18}{}$ (b) $\frac{9}{2} = \frac{108}{}$

(c) $\frac{11}{12} = \frac{}{24} = \frac{88}{}$ (d) $\frac{5}{8} = \frac{}{24} = \frac{35}{}$

(e) $\frac{4}{5} = \frac{36}{} = \frac{}{50}$ (f) $\frac{7}{6} = \frac{}{12} = \frac{35}{}$

(g) $\frac{3}{5} = \frac{}{15} = \frac{27}{}$

(h) $\frac{1}{2} = \frac{}{6} = \frac{}{14} = \frac{9}{} = \frac{10}{}$

(i) $\frac{3}{4} = \frac{}{20} = \frac{21}{}$

4. Pick the odd one out in the following.

(a) $\frac{1}{2}, \frac{3}{4}, \frac{5}{10}, \frac{51}{102}$ (b) $\frac{1}{4}, \frac{4}{8}, \frac{4}{16}, \frac{5}{20}, \frac{20}{80}$

(c) $\frac{1}{3}, \frac{11}{33}, \frac{13}{39}, \frac{14}{42}, \frac{15}{40}$ (d) $\frac{1}{7}, \frac{3}{21}, \frac{7}{49}, \frac{8}{55}, \frac{9}{63}$

5. Write the following mixed numbers in their simplest forms.

(a) $1\frac{3}{6}$ (b) $2\frac{5}{10}$ (c) $5\frac{6}{9}$ (d) $6\frac{18}{24}$

(e) $17\frac{45}{50}$ (f) $6\frac{32}{40}$ (g) $16\frac{2}{3}$

6. A fruit seller has one pumpkin and one melon. He cuts the pumpkin into eight equal parts and the melon into twelve equal parts. A customer wishes to buy three quarters of each. How many parts of each will she buy?

7. A carpenter has two types of timber. One type he cuts into six equal pieces, and the other into nine equal pieces. If he requires one third of each type to make some shelves, how many pieces will he use?

Ordering fractions by size

One way of comparing fractions by size involves first expressing them as fractions with a common denominator and then simply comparing the numerators. The one with a greater numerator is the greater fraction.

Example 6.7

Find the lowest common multiple of 8 and 12. Hence express the fractions $\frac{5}{8}$ and $\frac{7}{12}$ with a common denominator and determine which is the larger fraction.

Solution

$8 = 2^3$ $12 = 2^2 \times 3$

LCM of 8 and 12 is $2^3 \times 3 = 24$

$$\frac{5}{8} \xrightarrow[\times 3]{\times 3} \frac{15}{24}$$

$$\frac{7}{12} \xrightarrow[\times 2]{\times 2} \frac{14}{24}$$

Since 15 > 14 and the denominator is the same, then $\frac{15}{24} > \frac{14}{24}$.

$\therefore$ *$\frac{5}{8}$ is greater than $\frac{7}{12}$.*

Exercise 6.4

1. Which fraction is the greater in the pairs given below?

(a) $\frac{3}{13}, \frac{5}{13}$ (b) $\frac{11}{21}, \frac{9}{21}$ (c) $\frac{7}{16}, \frac{9}{16}$

(d) $\frac{5}{20}, \frac{3}{20}$

2. Express the fractions below with a common denominator.

(a) $\frac{7}{8}, \frac{2}{5}$ (b) $\frac{2}{3}, \frac{9}{10}$ (c) $\frac{1}{8}, \frac{3}{20}$

(d) $\frac{7}{12}, \frac{17}{2}$

3. Which fraction is the greater in the pairs given below?

(a) $\frac{10}{12}, \frac{11}{15}$ (b) $\frac{7}{8}, \frac{9}{10}$ (c) $\frac{7}{10}, \frac{9}{15}$

(d) $\frac{3}{4}, \frac{3}{5}$

4. Arrange the following fractions in ascending order (from the smallest to the largest).

(a) $\frac{3}{5}, \frac{1}{3}, \frac{4}{5}$ (b) $\frac{3}{8}, \frac{1}{5}, \frac{3}{5}$ (c) $\frac{6}{7}, \frac{1}{4}, \frac{5}{7}$

(d) $\frac{2}{3}, \frac{3}{5}, \frac{5}{6}, \frac{3}{10}$ (e) $\frac{3}{4}, \frac{3}{8}, \frac{5}{6}, \frac{2}{3}$

5. Business partners Kamau, Otieno and Mutua

shared their profit in the following manner. Kamau got $\frac{1}{3}$, Otieno got $\frac{1}{4}$, Mutua got $\frac{5}{12}$. Who received the greatest share?

- Mr. Mwangi wrote the following in his will:
  Eldest son to receive $\frac{3}{10}$ of his estate,
  youngest son to receive $\frac{1}{4}$ of his estate,
  daughter to receive $\frac{2}{5}$ of his estate.
  Who was to receive the smallest share.

- Write all the proper fractions whose denominators are 5 or less. Arrange them in descending order.

Addition and subtraction of fractions

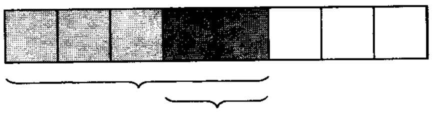
Fig 6-8 shows a whole unit divided into eight equal parts

Two parts of the whole are added to 3 parts.

$\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$

Fig. 6-8

In Fig. 6-9, 2 parts are taken away (i.e. subtracted) from 5 parts.



$\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$

Fig. 6-9

We see that:

> To add or subtract fractions which have a common denominator, we simply add or subtract the numerators.

e.g. 3 eighths + 2 eighths = 5 eighths, and
5 eighths – 2 eighths = 3 eighths

Example 6.8

Simplify $\frac{1}{6} + \frac{4}{9}$

Solution

Step 1: *Find the LCM of the denominators. The LCM of 6 and 9 is 36.*

Step 2: *Write each of the fractions with the LCM as the common denominator.*

$\frac{1}{6} = \frac{1\times 6}{6\times 6} = \frac{6}{36}$ *and*

$\frac{4}{9} = \frac{4\times 4}{9\times 4} = \frac{6}{36}$

Step 3: *Add the numerators and simplify.*

$$\frac{1}{6} + \frac{4}{9} = \frac{6}{36} + \frac{16}{36} = \frac{22}{36} = \frac{11}{18}$$

Example 6.9

Simplify $\frac{3}{5} - \frac{1}{2}$.

Solution

LCM of 5 and 2 is 10.

$\frac{3}{5} = \frac{6}{10}$ *and* $\frac{1}{2} = \frac{5}{10}$ *(Equivalent fractions with common denominator)*

$\frac{3}{5} - \frac{1}{2} = \frac{6}{10} - \frac{5}{10}$ *(Subtract numerators)*

$= \frac{1}{10}$

Example 6.10

Simplify (a) $2\frac{3}{4} + 1\frac{3}{8}$ *(b)* $8\frac{1}{3} - 2\frac{3}{4}$

Solution

(a) $2\frac{3}{4} + 1\frac{3}{8} = 2 + 1 + \frac{3}{4} + \frac{3}{8}$

$= 3 + \frac{3}{4} + \frac{3}{8}$

$= 3 + \frac{6}{8} + \frac{3}{8}$

$= 3 + \frac{9}{8} = 3 + \frac{8}{8} + \frac{1}{8}$

$= 3 + 1 + \frac{1}{8}$

$= 4\frac{1}{8}$

Alternative method

$2\frac{3}{4} = \frac{11}{4}$ *and* $1\frac{3}{8} = \frac{11}{8}$ *(changing mixed numbers into improper fractions)*

$2\frac{3}{4} + 1\frac{3}{8} = \frac{11}{4} + \frac{11}{8}$

$= \frac{22}{8} + \frac{11}{8}$ *(equivalent fractions with common denominators)*

$= \frac{33}{8}$ *(adding numerators)*

$= 4\frac{1}{8}$ *(changing to mixed numbers)*

(b) $8\frac{1}{3} - 2\frac{3}{4} = 8 - 2 + \frac{1}{3} - \frac{3}{4}$

$= 6 + \frac{4}{12} - \frac{9}{12}$

$= 5 + \frac{12}{12} + \frac{4}{12} - \frac{9}{12}$

$= 5 + \frac{16}{12} - \frac{9}{12}$

$= 5 + \frac{7}{12} = 5\frac{7}{12}$

Repeat this using the alternative method as in (a) above.

Exercise 6.5

1. Evaluate
   (a) $\frac{2}{8} + \frac{4}{8}$ (b) $\frac{1}{4} + \frac{3}{4}$ (c) $\frac{3}{6} + \frac{4}{6}$
   (d) $\frac{2}{8} + \frac{3}{8}$ (e) $\frac{4}{11} + \frac{5}{11}$ (f) $\frac{1}{24} + \frac{1}{24}$
   (g) $\frac{2}{15} + \frac{8}{15}$ (h) $\frac{9}{17} + \frac{3}{17}$ (i) $\frac{1}{6} + \frac{5}{6}$

2. Simplify
   (a) $\frac{5}{6} - \frac{1}{6}$ (b) $\frac{4}{5} - \frac{1}{5}$ (c) $\frac{13}{16} - \frac{11}{16}$
   (d) $\frac{31}{32} - \frac{18}{32}$ (e) $\frac{28}{64} - \frac{19}{64}$ (f) $\frac{7}{9} - \frac{5}{9}$
   (g) $\frac{5}{12} - \frac{1}{12}$ (h) $1 - \frac{2}{7}$ (i) $3 - \frac{5}{8}$

3. Simplify
   (a) $\frac{1}{4} + 3\frac{3}{4}$ (b) $2\frac{1}{7} + \frac{5}{7}$ (c) $5 + \frac{1}{7}$
   (d) $2\frac{5}{8} - \frac{3}{8}$ (e) $12\frac{9}{14} - \frac{2}{7}$ (f) $2\frac{7}{8} - 1\frac{3}{4}$

4. Evaluate
   (a) $\frac{2}{3} + \frac{1}{5}$ (b) $\frac{1}{6} + \frac{1}{5}$ (c) $\frac{1}{3} + \frac{3}{4}$
   (d) $\frac{5}{6} + \frac{7}{11}$ (e) $\frac{3}{10} + \frac{3}{4}$ (f) $\frac{1}{5} + \frac{6}{5} + \frac{3}{10}$
   (g) $\frac{1}{7} + \frac{2}{3} + \frac{7}{2}$ (h) $\frac{1}{7} + \frac{2}{3} + \frac{11}{5}$
   (i) $\frac{0}{3} + \frac{8}{9} + \frac{3}{4}$

5. Simplify
   (a) $\frac{1}{2} - \frac{1}{3}$ (b) $\frac{2}{3} - \frac{1}{6}$ (c) $\frac{9}{17} - \frac{9}{17}$
   (d) $\frac{2}{3} - \frac{2}{5}$ (e) $\frac{11}{12} - \frac{1}{2}$

6. Simplify
   (a) $1\frac{1}{2} + 2\frac{3}{5}$ (b) $2\frac{1}{3} + 4\frac{1}{2}$ (c) $1\frac{1}{5} + 2\frac{7}{8}$
   (d) $2\frac{3}{4} + 3\frac{1}{5}$ (e) $3\frac{1}{8} + 2\frac{4}{5}$

7. Rotich wants to make some shelves. He needs $3\frac{3}{4}$ m of wood for shelving and $1\frac{2}{3}$ m for the ends. How much wood should he buy?

8. On two consecutive days, a man painted $\frac{2}{5}$ and $\frac{1}{3}$ of a wall. What fraction remained to be painted?

9. What must be added to the sum of $5\frac{7}{8}$ and $4\frac{3}{8}$ to make 11?

10. A man left $\frac{1}{5}$ of his estate to his wife and $\frac{1}{3}$ to each of his two sons. The remainder was given to his daughter. What fraction of the estate did the daughter receive?

Multiplication of fractions

Multiplication of fractions by whole numbers

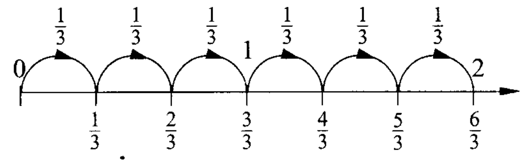
Consider

$$6 \times \frac{1}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$$

$$= \frac{1+1+1+1+1+1}{3}$$

$$= \frac{6}{3} = \frac{3}{3} + \frac{3}{3} = 1 + 1 = 2.$$

This multiplication can be illustrated on the number line as shown in Fig. 6-9.



$\therefore\ \frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=2$

Fig. 6-9

When working with fractions, the word 'of' means multiplication.

Consider $\frac{1}{3}$ of 6.

Given 6 items (Fig. 6-10(a)), they can be divided into 3 equal parts (Fig. 6-10(b)).

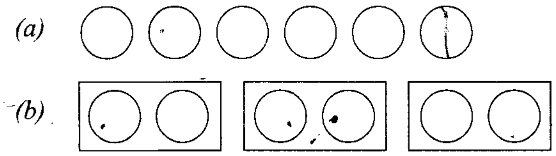


Fig. 6-10

$\frac{1}{3}$ of $6 = 2$ and $6 \times \frac{1}{3} = \frac{1}{3} \times 6 = 2$

$\therefore\ \frac{1}{3}$ of 6 means $\frac{1}{3} \times 6$, which is equivalent to dividing 6 by 3.

Thus:

> To multiply a fraction by a whole number we simply multiply the numerator by the whole number and then simplify the resulting fraction if necessary.

Multiplication of two fractions

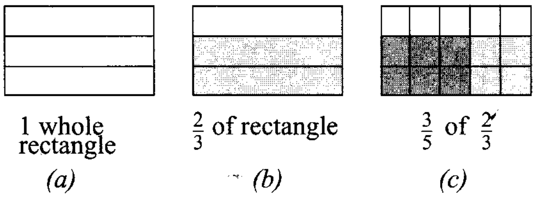


Fig. 6-11

Fig. 6-11(a) represents a whole rectangle.

In Fig. 6-11(b), the shaded portion represents $\frac{2}{3}$ of the rectangle.

Fig 6-11(c) shows a portion which has a dark shading. This represents $\frac{3}{5}$ of $\frac{2}{3}$. It represents 6 parts out of a total of 15.

$\therefore\ \frac{3}{5}$ of $\frac{2}{3} = \frac{6}{15} = \frac{6 \div 3}{15 \div 3} = \frac{2}{5}$.

Looking at this result, the answer could have been found as follows:

$\frac{3}{5} \times \frac{2}{3} = \frac{3 \times 2}{5 \times 3} = \frac{6}{15} = \frac{2}{5}$.

> To multiply fractions, multiply the numerators to get the numerator of the answer and multiply the denominators to get the denominator of the answer.
>
> e.g. $\frac{2}{7} \times \frac{1}{3} = \frac{2 \times 1}{7 \times 3} = \frac{2}{21}$.

Note that when multiplying a fraction by a whole number, the whole number can be treated as a fraction with denominator 1, e.g. $6 = \frac{6}{1}$.

Example 6.11

Simplify (a) $\frac{3}{4} \times \frac{2}{9}$ (b) $\frac{3}{8}$ *of* 72

(c) $\frac{9}{20} \times \frac{1}{12} \times \frac{10}{21}$

Solution

(a) $\frac{3}{4} \times \frac{2}{9} = \frac{3 \times 2}{4 \times 9} = \frac{6}{36} = \frac{1}{6}$.

(b) $\frac{3}{8}$ *of* $72 = \frac{3}{8} \times \frac{72}{1} = \frac{216}{8} = 27$.

(c) $\frac{9}{20} \times \frac{1}{12} \times \frac{10}{21} = \frac{9 \times 1 \times 10}{20 \times 12 \times 21}$

$= \frac{\overset{1\ \overset{3}{}}{\cancel{9}} \times 1 \times \overset{1}{\cancel{10}}}{\underset{2}{\cancel{20}} \times \underset{4}{\cancel{12}} \times \underset{7}{\cancel{21}}}$ *(To make work easier divide both numerator and denominator by the common factors as far as possible)*

$= \frac{1}{2 \times 4 \times 7}$

$= \frac{1}{56}$

Mixed numbers are multiplied by first writing them as improper fractions.

Example 6.12

Evaluate $3\frac{1}{2} \times 1\frac{1}{7} \times 1\frac{3}{10}$.

Solution

$3\frac{1}{2} \times 1\frac{1}{7} \times 1\frac{3}{10} = \frac{7}{2} \times \frac{8}{7} \times \frac{13}{10}$

$$= \frac{\cancel{7}^{1}\times\cancel{8}^{\cancel{4}^{2}}\times 13}{\cancel{2}_{1}\times\cancel{7}_{1}\times\cancel{10}_{5}}$$

$$= \frac{26}{5} = 5\tfrac{1}{5}.$$

Exercise 6.6

1. Evaluate
   (a) $\frac{3}{4}$ of 12 (b) $\frac{2}{5}$ of 75 (c) $\frac{7}{11}$ of 44
   (d) $\frac{3}{8}$ of 128 (e) $\frac{2}{3}$ of 54
2. Work out the following giving your answers in the simplest form.
   (a) $\frac{4}{5} \times \frac{6}{11}$ (b) $\frac{5}{18} \times \frac{9}{20}$ (c) $\frac{1}{2}$ of $\frac{8}{9}$
   (d) $\frac{12}{35} \times \frac{11}{15} \times \frac{21}{22}$
3. Work out the following and give your answers as mixed numbers in the simplest form.
   (a) $1\frac{2}{3} \times 5$ (b) $2\frac{1}{4} \times 8$
   (c) $4\frac{1}{2} \times 1\frac{3}{4} \times 1\frac{2}{5}$ (d) $2 \times 1\frac{7}{9}$
   (e) $\frac{3}{10} \times 4\frac{1}{6} \times \frac{8}{9}$
4. A certain recipe requires $\frac{2}{3}$ of a kilogram of meat. How much meat will be required to make $2\frac{1}{4}$ times the quantity?
5. Jane bought two packets of crisps each weighing $\frac{3}{8}$ of a kilogram. What mass of crisps did she buy?
6. Miss Njoroge's living room requires $2\frac{2}{3}$ tins of paint. Joe's living room is $1\frac{1}{8}$ times as big. How much paint of the same type will Joe's room require?
7. Julia visited her friend June who lives $\frac{4}{5}$ of 6 km away. She ran $\frac{2}{3}$ of the way. What distance does this represent? 3/4
8. In a particular girls' high school, $\frac{2}{5}$ of 200 form one students are the eldest in their families. How many students does this represent?
9. I used half a packet of milk for breakfast. John went home after school and drunk $\frac{5}{8}$ of what I left in the morning. How much milk remained?
10. By associating those values that work out more easily, evaluate the following:
    (a) $3 \times \frac{1}{5} \times \frac{1}{3} \times 5 \times 16$
    (b) $\frac{3}{5} \times 16 \times 6 \times \frac{5}{3} \times \frac{1}{2}$
    (c) $^-4 \times \frac{5}{8} \times {}^-3 \times 8$
    (d) $\frac{11}{7} \times {}^-\frac{6}{5} \times \frac{7}{11} \times 15 \times \frac{5}{6}$
    (e) $\frac{15}{4} \times {}^-5 \times {}^-4 \times \frac{8}{15} \times \frac{1}{5}$

Multiplication inverses

Study the products given below

$$\frac{3}{2} \times \frac{2}{3} = \frac{3 \times 2}{2 \times 3} = \frac{6}{6} = 1$$

$$\frac{1}{9} \times \frac{9}{1} = \frac{1 \times 9}{9 \times 1} = \frac{9}{9} = 1$$

$$\frac{2}{7} \times \frac{7}{2} = \frac{2 \times 7}{7 \times 2} = \frac{14}{14} = 1$$

For any non-zero number, there exists another non-zero number such that their product is 1. The two numbers are said to be **multiplication inverses** of each other.

Copy and complete Table 6-3 below.

Number	*Inverse*	*Product*
$\frac{2}{5}$	$\frac{5}{2}$	$\frac{2}{5} \times \frac{5}{2} = 1$
7	$\frac{1}{7}$	$7 \times \frac{}{1} = 1$
$\frac{3}{7}$	___	___ $= 1$
___	$\frac{8}{5}$	___ $= 1$
___	$-\frac{11}{12}$	___ $= 1$
$\frac{6}{11}$	___	___
___	$^-3\frac{1}{2}$	___

Table 6-3

Exercise 6.7

1. State the multiplication inverse of

   (a) $\frac{1}{2}$ (b) 6 (c) $-\frac{7}{8}$ (d) 4 (e) $\frac{9}{4}$

   (f) $-\frac{6}{11}$ (g) $\frac{5}{9}$ (h) 2 (i) $^-3$ (j) $-\frac{7}{15}$

   (k) $-\frac{8}{19}$ (l) $-\frac{1}{5}$ (m) 1 (n) $\frac{5}{-2}$ (o) $3\frac{1}{3}$

2. Find the missing numbers to make each of the following statements true.

   (a) $25 \times \square = 1$ (b) $-\frac{23}{24} \times \square = 1$

   (c) $^-5 \times \square = 1$ (d) $1 \times {}^-\frac{2}{9} = \square$

   (e) $13 \times \square = 1$ (f) $\frac{17}{5} \times \square = 1$

Division of fractions

In our work with integers, we saw that division is the reverse of multiplication. This fact is true even for fractions, so that:

$2 \times 3 = 6 \Rightarrow \quad 6 \div 2 = 3$

and $3 \times \frac{2}{5} = \frac{6}{5} \Rightarrow \quad \frac{6}{5} \div 3 = \frac{2}{5}$

Now, since $6 \times \frac{1}{2} = 3$ and $6 \div 2 = 3$ (as shown above), then

$6 \div \boxed{2} = 6 \times \boxed{\frac{1}{2}}$.

Similarly, since $\frac{6}{5} \times \frac{1}{3} = \frac{2}{5}$ and $\frac{6}{5} \div 3 = \frac{2}{5}$ (as shown above), then

$\frac{6}{5} \div \boxed{3} = \frac{6}{5} \times \boxed{\frac{1}{3}}$.

Also, $\frac{8}{3} \times \frac{3}{2} = 4$ and $\frac{8}{3} \div \frac{2}{3} = 4$. Thus,

$\frac{8}{3} \div \boxed{\frac{2}{3}} = \frac{8}{3} \times \boxed{\frac{3}{2}}$.

Now, state the missing fractions in each of the following:

$8 \div 2 = 8 \times \square \qquad \frac{3}{7} \div 4 = \frac{3}{7} \times \square$

$\frac{3}{7} \div \frac{2}{3} = \frac{3}{7} \times \square$

> Note that $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{3}{2}$ are multiplication inverses of 2, 3 and $\frac{2}{3}$ respectively. This shows that dividing by a fraction is the same as **multiplying by the multiplication inverse** of the divisor.

Note that when working with mixed numbers, they must first be changed into improper fractions.

Example 6.13

Evaluate (a) $\frac{3}{7} \div \frac{4}{5}$ (b) $2\frac{7}{8} \div \frac{1}{4}$

Solution

(a) $\frac{3}{7} \div \frac{4}{5} = \frac{3}{7} \times \frac{5}{4} = \frac{3 \times 5}{7 \times 4} = \frac{15}{28}$

(b) $2\frac{7}{8} \div \frac{1}{4} = \frac{23}{8} \times \frac{4}{1} = \frac{23 \times 4}{8 \times 1} = \frac{23}{2} = 11\frac{1}{2}$

Exercise 6.8

1. Simplify:

   (a) $\frac{3}{4} \div \frac{2}{3}$ (b) $4\frac{1}{2} \div \frac{5}{9}$ (c) $4\frac{1}{5} \div \frac{7}{8}$

   (d) $8\frac{2}{3} \div \frac{7}{12}$ (e) $1\frac{3}{4} \div \frac{1}{4}$ (f) $3\frac{1}{3} \div {}^-2$

   (g) $4\frac{2}{3} \div 3$ (h) $\frac{3}{4} \div \frac{1}{16}$

2. Re-write each of the following, inserting the missing sign so that the statement is true.

   (a) $\frac{1}{3} \square \frac{1}{3} = 1$ (b) $\frac{1}{6} \square \frac{1}{6} = 0$

   (c) $\frac{1}{2} \square \frac{1}{2} = \frac{1}{4}$ (d) $5 \square \frac{1}{4} = 1\frac{1}{4}$

   (e) $1 \square \frac{1}{4} = 4$ (f) $\frac{1}{2} \square \frac{1}{3} = \frac{5}{6}$

   (g) $\frac{4}{5} \square {}^-\frac{4}{5} = {}^-1$ (h) $\frac{4}{3} \square \frac{3}{4} = \frac{16}{9}$

   (i) $\frac{4}{3} \square \frac{3}{4} = \frac{7}{12}$ (j) $\frac{4}{3} \square \frac{3}{4} = \frac{25}{12}$

3. Find the missing numbers.

   (a) $-\frac{2}{3} + \square = {}^-1$ (b) $-\frac{2}{5} + 9 = \square$

   (c) $5 + \square = -\frac{3}{4}$ (d) $\square + {}^-\frac{2}{3} = 0$

   (e) $\frac{2}{3} \times \square = {}^-1$ (f) $\frac{4}{5} - \square = -\frac{1}{5}$

   (g) $3 \times \square = \frac{6}{4}$ (h) $\frac{3}{4} - \square = \frac{1}{2}$

   (i) $-\frac{2}{3} - \square = -\frac{1}{3}$ (j) $\frac{12}{17} + \square = 2$

   (k) $-\frac{1}{3} \times \square = \frac{1}{2}$ (l) $-\frac{5}{6} \times \square = \frac{4}{5}$

4. Evaluate:

   (a) $\frac{3}{-7} + \frac{5}{-3}$ (b) $\frac{2}{3} + \frac{-3}{10}$ (c) $\frac{0}{5} - \frac{-2}{-3}$

(d) $\frac{8}{^-4} - \frac{2}{^-3}$ (e) $\frac{3}{8} - (\frac{^-5}{2})$

5. Find the value of:

   (a) $\frac{1}{3}$ of 60 minutes (b) $\frac{1}{6}$ of 24 hours

   (c) $\frac{5}{8}$ of 100 cm (d) $\frac{3}{5}$ of 1 000 g

   (d) $\frac{2}{7}$ of 7 days

6. Jane ate $\frac{1}{4}$ of her cake in the morning, $\frac{1}{3}$ of the remainder in the afternoon, and $\frac{1}{2}$ of the remainder in the evening. How much cake remained for the next day?

7. Three quarters of houses in Matopeni Estate have electricity. Half of those with electricity have television sets. If 60 houses have television sets, how many houses are there in the estate?

8. In 3 hours a lighted candle shrinks by $\frac{2}{3}$ of its original length. What fraction burns out in 1 hour? For how much longer will the remaining part last?

9. The length of a rectangle is $3\frac{1}{2}$ cm and its breadth is $2\frac{7}{8}$ cm. Find

   (a) the sum of all the sides.

   (b) the product of the length and the breadth.

10. In a 1996 by-election, $\frac{1}{10}$ of the registered voters in Githioro Constituency did not vote, $\frac{1}{5}$ voted for Mr. Rono, and the rest voted for Mrs. Wafula. If Mrs. Wafula received 3 500 votes, how many registered voters were in that constituency?

Order of fractions

The rules that govern operations on integers apply for fractions as well.

1. If there are brackets, perform the operations within them first.
2. If there is 'of', it must be performed before any other operation.
3. Division should be done next followed by multiplication, or working from left, perform whichever comes first.
4. Addition and subtraction follow in that order, or working from the left, perform whichever comes first.

This can be easily remembered using the abbreviation **BODMAS** (**B**rackets, **O**f, **D**ivision, **M**ultiplication, **A**ddition, **S**ubtraction).

Example 6.14

Simplify $\frac{3}{5} \div \frac{2}{3} - \frac{1}{6} \times \frac{7}{12}$ *of* $(\frac{1}{2} + \frac{4}{5})$.

Solution

$\frac{3}{5} \div \frac{2}{3} - \frac{1}{6} \times \frac{7}{12}$ *of* $(\frac{1}{2} + \frac{4}{5})$.

$= \frac{3}{5} \div \frac{2}{3} - \frac{1}{6} \times \frac{7}{12}$ *of* $\frac{13}{10}$ *(brackets first)*

$= \frac{3}{5} \div \frac{2}{3} - \frac{1}{6} \times \frac{7}{12} \times \frac{13}{10}$ *('of' next)*

$= \frac{3}{5} \times \frac{3}{2} - \frac{1}{6} \times \frac{7}{12} \times \frac{13}{10}$ *(division next)*

$= \frac{9}{10} - \frac{91}{720}$ *(multiplication next)*

$= \frac{648 - 91}{720} = \frac{557}{720}$ *(finally subtraction)*

Exercise 6.9

Simplify

1. $2\frac{1}{2} + 3\frac{1}{3} \div 1\frac{3}{4}$
2. $\frac{5}{6}$ of $(3 - \frac{2}{3})$
3. $1\frac{4}{5} - (\frac{3}{4} \div \frac{5}{18}) + \frac{14}{15}$
4. $\dfrac{3\frac{1}{2} + 2\frac{1}{4}}{3\frac{1}{2} - 2\frac{1}{4}}$
5. $(\frac{1}{2} + \frac{6}{7} \times \frac{5}{18}) \div (\frac{4}{5} - \frac{1}{6} \div \frac{3}{14})$
6. $\frac{1}{2}$ of $\frac{2}{5} \div \frac{3}{15} + \frac{7}{10} \times \frac{6}{7} - (1\frac{1}{2} \times \frac{1}{3})$
7. $\frac{-9}{5} \times \frac{27}{35} \times \frac{^-5}{9} \times \frac{^-35}{27}$
8. $^-\frac{1}{5} \div (\frac{2 \times 4}{1 - 6}) \times \frac{8}{^-5}$
9. $\frac{5}{3} \times \frac{6}{10} \div \frac{^-5}{4} - \frac{1}{4}$
10. $\dfrac{\frac{2}{3} \times \frac{5}{7} \div \frac{1}{18}}{\frac{4}{5} \times 6 \div 1\frac{7}{12}}$

Problems involving fractions

This section provides further practice on solving problems involving fractions. The exercise emphasises more on word problems involving fractions in real life situations.

Exercise 6.10

1. Find the excess of $7\frac{8}{9}$ over $3\frac{11}{12}$.
2. By what numbers must $3\frac{1}{2}$ be multiplied to give $6\frac{3}{4}$?
3. The product of two numbers is $4\frac{2}{5}$ and one of them is $3\frac{1}{7}$. Find
   (a) their sum.
   (b) their difference.
4. Simplify
   (a) $2\frac{1}{3} + 3\frac{2}{5}$ (b) $3\frac{2}{5} - 2\frac{1}{3}$
   (c) $\dfrac{3 \times 3\frac{2}{5} - 2 \times 2\frac{1}{3}}{3\frac{2}{5} + 2 \times 2\frac{1}{3}}$
5. A sum of K£ 920 was shared among Jane, Beth and Kate. Jane got one fifth and Beth got one eighth of the sum. How much did each of the three get?
6. Find the difference between one fifth of 420 and one seventh of the same number.
7. A water tank is $\frac{3}{7}$ full, and after adding 52 litres, it is $\frac{4}{5}$ full. What is its total capacity?
8. In a school, three sevenths of the pupils are boys, and the rest are girls. Three quarters of the boys and eleven twelfths of the girls passed their annual examination. If 52 pupils failed to pass, find the number of pupils in the school.
9. One third of a pole was in mud, three seventh of the remainder in water and the rest measuring 12 m above the water. Find the length of the pole.
10. Mrs. Otieno spends $\frac{2}{5}$ of her daily earnings on groceries and $\frac{2}{7}$ on bread and milk. If she earns Sh 350 every day, how much does she save?
11. A reel contains 300 m of thread. How many pieces, each measuring $2\frac{5}{8}$ m, can be cut from it and what length is left over?
12. A section of a road is 3 080 m long. One fifth of it is earth road and $\frac{1}{7}$ is tarmac. Find the difference in length between the two stretches.
13. A quarter of a field is planted with potatoes, $\frac{4}{7}$ of the remainder with maize, and the remaining part with greens. Find the fraction that is planted with greens.
14. My car's petrol tank was $\frac{5}{8}$ full when I left home for school. By the time I got to school, the tank was $\frac{2}{8}$ full. How much more petrol do I need to get back home?
15. Chebet can do a piece of work in 4 hours. Cheboiyo can do the same job in 5 hours and Chemai can do the same job in 12 hours. What fraction of the job can each do in 1 hour. If the three men work together for one hour, what fraction of the job would ~~they~~ do?