7 DECIMAL FRACTIONS

What is a decimal?

When a fraction is written with a power of 10 as its denominator, it is called a **decimal fraction** or simply a **decimal**.

For example:

$\frac{1}{10} = 0.1$; $\frac{2}{100} = 0.02$; $1\frac{3}{1\,000} = 1.003$; $\frac{4}{10\,000} = 0.000\,4$; $23\frac{5}{100\,000} = 23.000\,05$; etc.

The dot is called the decimal point and it is used to separate whole numbers from the fractional parts.
A digit after the decimal point is said to stand in a certain decimal place.

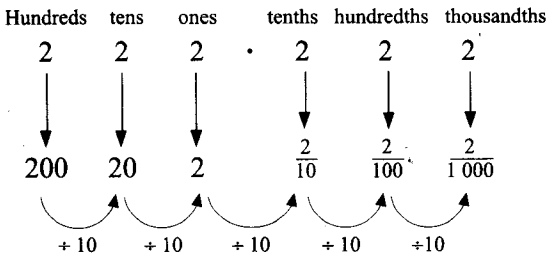
0.1 is read as 'zero point one' and it means one tenth.

0.02 is read as 'zero point zero two' and it means two hundredths.

0.003 is read as 'zero point zero zero three' and it means three thousandths.

4.0905 is read as 'four point zero nine zero five' and it means four, nine hundredths and five ten thousandths.

In the number system that we use, each digit of a number has ten times the value of the digit immediately to its right, e.g.



As we move to the right, each place value column is one tenth of the previous one. The place of the digit in a decimal number gives its value just as in whole numbers.

	Whole number parts				Fractional parts		
	Hundreds (H)	Tens (T)	Units or Ones (U)	•	tenths (t)	hundredths (h)	thousandths (th)
(a)	1	2	4	.	3	0	5
(b)		3	1	.	0	3	1
(c)			5	.	0	0	6

Table 7-1

In Table 7-1,

(a) means one hundred twenty four, three tenths and five thousandths,

(b) means thirty one, three hundredths and one thousandths, and

(c) means five ones and six thousandths.

Each one of the above numbers is given to three decimal places.

Example 7.1

Write the numbers below in decimal form as shown in Table 7-1.

(a) $\frac{4}{10}$ *(b)* $\frac{5}{100}$ *(c)* $281 + \frac{5}{10} + \frac{6}{100} + \frac{9}{1\,000}$

Solution

Table 7-2 shows the numbers in decimal form.

	H	*T*	*U*	.	*t*	*h*	*th*
(a)			*0*	.	*4*		
(b)			*0*	.	*0*	*5*	
(c)	*2*	*8*	*1*	.	*5*	*6*	*9*

Table 7-2

Note that these numbers are written without place value names as:

$\frac{4}{10} = 0.4$, $\frac{5}{100} = 0.05$ and

$281 + \frac{5}{10} + \frac{6}{100} + \frac{9}{1\,000} = 281.569$.

Example 7.2

Write $2 \times 10 + 5 \times 1 + 3 \times \frac{1}{10} + 7 \times \frac{1}{100}$ in decimal form and state the number of decimal places in the number.

Solution

$2 \times 10 + 5 \times 1 + 3 \times \frac{1}{10} + 7 \times \frac{1}{100}$

$= 20 + 5 + 0.3 + 0.07$

$= 25.37$

The number has two decimal places.

Exercise 7.1

1. Make a table of place values and show in it the following numbers.
   (a) $10 + \frac{1}{10} + \frac{2}{100} + \frac{7}{1\,000}$ (b) $5 + \frac{2}{1\,000}$
   (c) $8 + \frac{1}{10} + \frac{1}{100} + \frac{6}{1\,000}$
   (d) $200 + 50 + 5 + \frac{5}{10} + \frac{5}{100} + \frac{5}{1\,000}$
   (e) $\frac{3}{10} + \frac{7}{100} + \frac{1}{1\,000}$
   (f) $400 + 40 + 4 + \frac{4}{100} + \frac{4}{1\,000}$

2. Write the following decimal numbers in words, e.g. 24.2 means 'Two tens four ones and two tenths'.

	H	T	U	.	t	h	th
(a)		2	4	.	2		
(b)			7	.	6	7	8
(c)			0	.	0	5	4
(d)	4	5	0	.	4	0	4
(e)	2	0	3	.	8	1	6
(f)	6	0	0	.	0	1	

Table 7-3

3. Write the following in decimal form.
   (a) Seven tenths
   (b) Two tenths, six hundredths
   (c) Twelve and seven thousandths
   (d) Four hundred, three thousandths

4. Using a table, give the place value of each digit in the following.
   (a) 0.03 (b) 0.005 (c) 0.32
   (d) 0.093 (e) 2.371 (f) 19.034
   (g) 8.9873 (h) 806.41

5. State the number of decimal places in each of the following numbers.
   (a) 0.045 (b) 0.0028
   (c) 5.5504 (d) 8.40013

6. Express the following in decimal form.
   (a) $4 \times 10 + 6 \times \frac{1}{10} + 2 \times \frac{1}{10^2}$
   (b) $5 \times 10^2 + 3 \times 10 + 2 \times 1 + 5 \times \frac{1}{10} + 7 \times \frac{1}{10^2}$

Addition and subtraction of decimals

When adding or subtracting decimals, it is best to work vertically so as to keep the digits in their appropriate place value columns and the decimal points under each other. Then proceed as for whole numbers.

Example 7.3

Evaluate (a) 0.68 + 0.59 (b) 6.85 + 198.4

Solution

The working may be laid out as in Table 7-4 (a) or as in part (b) without the place value table.

(a)

U	.	*t*	*h*
0	.	*6*	*8*
0	.	*5*	*9*
1	.	*2*	*7*

Table 7-4

(b)
$$\begin{array}{r} 6.85 \\ 198.40 \\ \hline 205.25 \end{array} \; +$$

Example 7.4

Evaluate: (a) 0.963 – 0.25 (b) 614 – 38.18
(c) 0.53 – 1.72

Solution

The working may be done using a place value table as in (a) (Table 7-5) or using vertical format as in parts (b) and (c).

(a)

U	.	*t*	*h*	*th*
0	.	*9*	*6*	*3*
0	.	*2*	*5*	*0*
0	.	*7*	*1*	*3*

–

Table 7-5

(b)
$$\begin{array}{r} 614.00 \\ 38.18 \\ \hline 575.82 \end{array} \; -$$

(c) Just like with integers, it is possible to subtract a larger decimal from a smaller one.

Compare 0.53 – 1.72 with 3 – 5.

$3 - 5 = -(5 - 3) = {}^{-}2$

Similarly, 0.53 – 1.72 = – (1.72 – 0.53), i.e. subtract the smaller number from the larger one and then change the sign.

This working may be laid out in vertical format as follows.

$$\begin{array}{r} 1.72 \\ 0.53 \\ \hline 1.19 \end{array} \; -$$

$\therefore \; 0.53 - 1.72 = {}^{-}1.19$

Exercise 7.2

1. Evaluate
(a) 0.71 + 0.84
(b) 0.721 + 0.55
(c) 4.58 + 3.67
(d) 458.62 + 32.47 + 3.259
(e) 18.6 + 0.274 9 + 8.506

2. Evaluate
(a) 0.7 – 0.13 (b) 16.14 – 7.318 5
(c) 4.96 – 1.844 (d) 6.003 – 5.162

3. Evaluate
(a) 0.71 – 0.84 (b) 0.85 – 3.297
(c) 1.015 – 2.34 (d) 0.005 2 – 98.39

4. Evaluate
(a) 9.421 8 – 6.345 6 + 3.12
(b) 6.950 + 8.432 – 3.010 4

5. A rectangle measures 3.34 cm by 2.88 cm. Find:
(a) the total distance round the rectangle,
(b) the difference between the length and the breadth.

6. Copy and complete the following patterns of decimals.
(a) 0.1, 0.2, 0.3, ___, ___, ___, ___
(b) 0.75, 0.76, 0.77, ___, ___, ___, ___
(c) 0.52, ___, 0.56, 0.58, ___, ___, ___
(d) 0.845, 0.849, ___, ___, ___, ___

7. Write the numbers which are one tenth less than each of the following numbers.
(a) 0.65 (b) 0.41
(c) 0.835 (d) 9.82

8. What is three hundredths more than
(a) 0.866 (b) 0.059
(c) 0.829 2 (d) 0.846

Multiplication and division of decimals

Multiplication and division of decimals by powers of 10

Consider the following.

$65 \times 10 = 650$

$65 \times 100 = 6\,500$

$65 \times 1\,000 = 65\,000.$

Note that 65 is the same as 65.0. So, in each case, the decimal point has moved to the right as many places as there are zeros in the multiplier.

Similarly;

$0.65 \times 10 = 6.5$

$0.65 \times 100 = 65$

$0.65 \times 1\,000 = 650.$

Recall that division is the reverse of multiplication. We have just seen that when multiplying decimals with powers of 10, the decimal point moves to the right. In division, therefore, the decimal point moves to the left as many places as there are zeros in the divisor. For example,

$6\,500 \div 10 = 650$

$6\,500 \div 100 = 65$

$6\,500 \div 1\,000 = 6.5$

$6\,500 \div 10\,000 = 0.65$

Example 7.5

Evaluate (a) $5.21 \div 10$

(b) $5.21 \div 1\,000$

Solution

(a) $5.21 \div 10 = 0.521$

(b) $5.21 \div 1\,000 = 0.005\,21$

Exercise 7.3

1. Evaluate
   (a) 5.86×10 (b) 143.8×100
   (c) $0.169\,3 \times 1\,000$ (d) $0.000\,78 \times 100$
2. Evaluate
   (a) $4.396 \div 100$ (b) $4.9 \div 10$
   (c) $22.1 \div 10\,000$ (d) $690 \div 1\,000$
3. Find the missing number in each of the following.
   (a) $8.71 \times \_\_\_ = 871$
   (b) $0.505\,01 \times \_\_\_ = 50.501$
   (c) $0.716\,3 \times \_\_\_ = 71\,630$
   (d) $78.5 \div \_\_\_ = 0.007\,85$
   (e) $107 \div \_\_\_ = 0.010\,7$
4. Multiply each of the following numbers by 10, 100, 1 000.
   (a) 0.113 (b) 0.001 2 (c) 18.001
5. Divide each of the following numbers by 10, 100, 100 000.
   (a) 71.6 (b) 1 005.1 (c) 17.015
6. Write down the next three numbers in the number pattern 10 000, 1 000, 100, ...

Standard form of numbers

Consider the following patterns of powers of 10.

$100\,000 = 10 \times 10 \times 10 \times 10 \times 10 = 10^5$

$10\,000 = 10 \times 10 \times 10 \times 10 = 10^4$

$1\,000 = 10 \times 10 \times 10 = 10^3$

$100 = 10 \times 10 = 10^2$

$10 = 10 = 10^1$

$1 = 1 = 10^0$

$0.1 = \frac{1}{10} = \frac{1}{10^1} = 10^{-1}$

$0.01 = \frac{1}{10 \times 10} = \frac{1}{10^2} = 10^{-2}$

$0.001 = \frac{1}{10 \times 10 \times 10} = \frac{1}{10^3} = 10^{-3},$

and so on.

This pattern involves successive division by 10, as we move from top downwards.

When written in power form, the powers decrease in steps of 1 as we move from top downwards.

Note that for numbers less than 1 (i.e. $0.1 = \frac{1}{10^1} = 10^{-1}$, $0.01 = \frac{1}{10^2} = 10^{-2}$, $0.001 = \frac{1}{10^3} = 10^{-3}$, etc.), the power of 10 is negative.

Now consider the following numbers.

$$3\,750 = 3.75 \times 1\,000 = 3.75 \times 10^3$$
$$375 = 3.75 \times 100 = 3.75 \times 10^2$$
$$37.5 = 3.75 \times 10 = 3.75 \times 10^1$$
$$3.75 = 3.75 \times 1 = 3.75 \times 10^0$$
$$0.375 = 3.75 \times \frac{1}{10} = 3.75 \times 10^{-1}$$
$$0.037\,5 = 3.75 \times \frac{1}{100} = 3.75 \times 10^{-2},$$

and so on.

Each of these numbers is said to have been written in **standard** or **scientific form**.

Any number can be written in this form.

> A number is said to be written in **standard** or **scientific form** if it is written as $\mathbf{A \times 10^n}$, where $1 \leq A < 10$ and n is an integer.

In every day life, people use phrases such as '4.5 million', '3.21 billion', etc. In reality, they are speaking using standard form since:

$$4.5 \text{ million} = 4.5 \times 1\,000\,000 = 4.5 \times 10^6$$
$$3.21 \text{ billion} = 3.21 \times 1\,000\,000\,000 = 3.21 \times 10^9, \text{ etc}$$

However, a number such as 82.3 million is not in standard form since 82.3 is **not** between 1 and 10. But, it can easily be expressed in standard form as follows.

$$82.3 \text{ million} = 82.3 \times 1\,000\,000 = 82\,300\,000 = 8.23 \times 10^7$$

or

$$82.3 \text{ million} = 82.3 \times 1\,000\,000 = 8.23 \times 10 \times 1\,000\,000 = 8.23 \times 10^7$$

Similarly, when one talks of a computer having a memory of 32 Kilobytes or a backing storage of 64 Megabytes, he/she is stating the size of the memory or backing storage in a form that is close to standard form, since 'kilo' means 1 000 and 'Mega' means 1 000 000. Other prefices used with computers are 'Giga' and 'Tera', which means 1 000 000 000 and 1 000 000 000 000 respectively.

In standard form,

32 Kilobytes = 3.2×10^4 bytes

64 Megabytes = 6.4×10^7 bytes.

State 5 Gigabytes and 23 Terabytes in standard form.

Exercise 7.4

1. Write each of the following in standard form.

(a) 750 (b) 5 100 (c) 28 000 000

(d) 49 000 000 (e) 821 500 000

2. Write each of the following in standard form.

(a) 0.43 (b) 0.008 7 (c) 0.000 64

(d) 0.000 003 2 (e) 0.000 000 081

3. Which of the following are not in standard form?

(a) 3.8×10^7 (b) 25×10^5

(c) 13×10^6 (d) 7.8×3^5

4. Write the following as ordinary numbers.

(a) 3.9×10^6 (b) 2.4×10^{-3}

(c) 4.31×10^7 (d) 3.8×10^{-5}

5. Each of the following represents a number and its equivalent standard form. Copy and fill in the gaps.

(a) $0.000\,045 = 4.5 \times 10^{\square}$

(b) $\square = 7.21 \times 10^6$

(c) $78\,634 = \square \times \square$

(d) $\square \times 10^{\square} = 0.005\,45$

6. Write the following in standard form.
   (a) 6.43 (b) 268.5
   (c) 6 829.54 (d) 0.02×10^3

7. Write the following in standard form.
   (a) Five thousand (b) 93 million
   (c) 21.6 million (d) 473.9 billion

8. Express the following in standard form.
   (a) 6 500 km, the radius of the earth.
   (b) 576 000 km^2, the area of Kenya.
   (c) 148.8 million km, the approximate distance of the sun from the earth.
   (d) 0.000 000 000 1 mm, the approximate diameter of an atom.

Multiplication of decimals

Consider 0.25×0.3

$$0.25 \times 0.3 = \frac{25}{100} \times \frac{3}{10} = \frac{75}{1\,000} = 0.075$$

This could be done as follows:

(i) Temporarily ignore the decimal points and multiply the numbers:
$25 \times 3 = 75$

(ii) In the product, beginning from the right, count off as many decimal places as there are in the two numbers combined. In 0.25, there are 2 decimal places and in 0.3 there is 1 decimal place, a total of 3.

(iii) Insert the decimal point as is appropriate
$0.25 \times 0.3 = 0.075$

Example 7.6

Evaluate (a) 0.04×3 (b) 6.08×0.4

Solution

(a) 0.04×3

$4 \times 3 = 12$

$0.04 \times 3 = 0.12$

2 decimal places + zero decimal places = 2 decimal places

(b) 6.08×0.4

$608 \times 4 = 2\,432$

$6.08 \times 0.4 = 2.432$

2 decimal places + one decimal places = 3 decimal places

Exercise 7.5

1. Calculate
   (a) 0.6×8 (b) 0.04×6
   (c) 4.021×7 (d) 0.415×3
   (e) 10.007×7

2. Evaluate
   (a) 12.76×0.04 (b) 143.8×0.03
   (c) 2.005×1.3 (d) 0.452×0.53

3. Without multiplying out, state the number of decimal places in each of the following products.
   (a) 10.9×0.7 (b) 6.007×2.1
   (c) 6.08×0.04 (d) 0.006×0.09
   (e) 6.13×0.08 (f) 7.006×0.2
   (g) 0.82×0.4

4. Evaluate
   (a) 125.6×0.07 (b) 184.3×0.39
   (c) $0.004\,1 \times 0.092$ (d) 6.13×0.18
   (e) 2.89×0.024 (f) 28.7×0.025

5. A tailor bought 3.5 m of material at Sh 250.50 per metre. How much did she spend?

6. One hectare of land costs Sh 150 000. Find the cost of 2.7 hectares.

7. Six adults travel by bus from Nairobi to Mombasa at a fare of K£ 25.68 per person. How much does it cost the six men to travel to Mombasa and back?

8. Given that 1 cm^3 of sea water has a mass of 1.026 g, find the mass of 18.44 cm^3 of the same water.

9. Find the total height of a pile of 23 books if the thickness of each book is 3.4 cm.

Division of decimals

If the divisor is a whole number, divide as for whole numbers, but make sure that the decimal point in the answer is vertically aligned with that of the dividend (Example 7.7)

Example 7.7

Divide 1.69 by 13

Solution

$$\begin{array}{r} 0.13 \\ 13\overline{)1.69} \\ -1\,3 \\ \hline 39 \\ -39 \\ \hline \cdot\cdot \end{array}$$

$1.69 \div 13 = 0.13$

If the divisor is a decimal, multiply both the divided and the divisor by the smallest power of 10 which makes the divisor a whole number. Then proceed as before.

Example 7.8

Evaluate $8.45 \div 0.005$

Solution

$$\frac{8.45}{0.005} = \frac{8.45 \times 1\,000}{0.005 \times 1\,000} = \frac{8\,450}{5} = 1\,690$$

In problems where multiplication and division are combined, the idea of cancellation may be used. This is easily done if the denominator and numerator are both multiplied by the same power of 10 to make at least the denominator a whole number.

Example 7.9

Simplify $\dfrac{0.016\,5 \times 12.75}{0.25 \times 0.007\,5}$

Solution

$$\frac{0.016\,5 \times 12.75}{0.25 \times 0.007\,5}$$

$$= \frac{0.016\,5 \times 12.75 \times 100 \times 10\,000}{0.25 \times 0.007\,5 \times 100 \times 10\,000}$$

$$= \frac{\overset{11}{\cancel{33}} \times \overset{51}{\cancel{255}}}{\underset{1}{\cancel{25}} \times \underset{5}{\cancel{75}}}$$

$$= \frac{11 \times 51}{5}$$

$$= \frac{561}{5}$$

$$= 112.2$$

Exercise 7.6

1. Evaluate
   (a) $153 \div 8$ (b) $0.068 \div 7$
   (c) $5.7 \div 11$ (d) $172.543 \div 4$

2. Evaluate
   (a) $0.64 \div 0.2$ (b) $0.737 \div 1.1$
   (c) $0.84 \div 0.6$ (d) $6.66 \div 9$
   (e) $0.054\,6 \div 0.12$

3. Evaluate
   (a) $14.28 \div 3.4$ (b) $11.5 \div 4.6$
   (c) $4.6 \div 11.5$

4. Find the missing number in $\frac{\quad}{0.004} = 2.84$

5. Evaluate $\frac{0.321\,4 \times 0.025}{0.8 \times 0.05}$

6. Simplify the following.
   (a) $\frac{0.187\,5 \times 3.24}{0.067\,5}$ (b) $\frac{0.135 \times 25.5}{0.0187\,5}$
   (c) $\frac{0.011\,4 \times 0.079\,1}{0.006\,78}$ (d) $\frac{0.062\,5}{1.6 \times 0.125}$

7. A six-hour school day is divided into eight equal lessons. How long does each lesson last?

8. Mama Kazi is paid K£ 18 for a five-hour day. How much does she earn per hour?

9. Five litres of water have a mass of 5.025 kg. Find the mass of 2 litres of water.

10. If 4 kg of meat cost a total of K£ 6.36, find the cost of 7 kg of meat.

Multiplication and division of decimals by multiples of 10.

To multiply a decimal by a multiple of 10, e.g. 20, 30, 40, etc, multilpy by the non-zero part, then move the decimal point one place to the right for each zero.

Example 7.10

Evaluate $0.37 \times 4\,000$

Solution

$0.37 \times 4\,000 = 0.37 \times 4 \times 1\,000$
$= 1.48 \times 1\,000$
$= 1\,480$

To divide by numbers such as 20, 30, 40, 50, etc, divide first by the non-zero part, then move the decimal point one place to the left for each zero.

Example 7.11

Evaluate $158.4 \div 4$

Solution

$158.4 \div 4 = 39.6$ *(First divide by 4)*
$39.6 \div 100 = 0.396$ *(Move decimal point two places to the left)*
$\therefore\ 158.4 \div 400 = 0.396$

Note: Where there are mixed operations in a problem on decimals, the order of operations should be the same as was established for integers and fractions, i.e. BODMAS.

Exercise 7.7

Evaluate the products of questions 1–8.

1. 4.5×20
2. $0.035\,06 \times 50$
3. 42.56×30
4. $0.761 \times 4\,000$
5. 791.3×300
6. 842.7×60
7. 73.4×170
8. $51.4 \times 20 \times 110$
9. Find the cost of 8.5 m of linen at Sh 700 per metre.
10. Multiply 5.72 by:
    (a) 50 (b) 20 (c) 120 (d) 130
11. Work out
    (a) $0.14 \times 3.25 + 213.6 \div 3.2$
    (b) $4.63 \times 0.85 \div 0.005$

(c) $6.3 - 406 \times 2.9 + 5.7$

(d) $\dfrac{0.008 - 0.0734}{10.4 + 0.5}$

(e) $\dfrac{0.37 \times 20.1 + 0.03}{0.08}$

(f) $0.04\,(0.008 - 1.205 + 4.375) \div 0.08$

Rounding off

Rounding off to a number of decimal places

Consider the following case:

The length of a line segment is stated as 15.695 cm.

It is highly unlikely that a measurement can be taken to the nearest thousandth of a centimetre. For all practical purposes, this length can be stated as 15.7 cm (to the nearest tenth or to 1 decimal place (1 d.p)). We say that the length has been **rounded off** to the nearest tenth or correct to 1 decimal place.

Example 7.12

Write the number 7.85263 correct to

(a) 4 d.p (b) 3 d.p (c) 2 d.p (d) 1 d.p
(e) the nearest whole number.

Solution

(a) 7.85263 ≈ 7.8526 (to 4 d.p)
(b) 7.85263 ≈ 7.853 (to 3 d.p)
(c) 7.85263 ≈ 7.85 (to 2 d.p)
(d) 7.85263 ≈ 7.9 (to 1 d.p)
(e) 7.85263 ≈ 8 (to the nearest whole number)

In writing a number correct to a given number of decimal places, count from the first digit after the decimal point.

(a) If the first digit to be omitted is a digit from 5 to 9, add 1 to the last digit retained (i.e. round up the number).

(b) If the first digit to be omitted is 4 or less, then the last digit retained remains unaltered (i.e. round down the number).

Rounding off to a number of significant figures

Consider the number 652.73.

1. $652.73 \approx 700$ since 652.73 is closer to 700 than 600.
2. $652.73 \approx 650$ since 652.73 is closer to 650 than to 660.
3. $652.73 \approx 653$ since 652.73 is closer to 653 than 652.
4. $652.73 \approx 652.7$ since 652.73 is closer to 652.7 than to 652.8.

In 1, **importance** (**significance**) is given to only the hundreds place value, so that $652.73 = 700$ to 1 significant figure.

In 2, significance is given to both the hundreds and tens place values, so that $652.73 = 650$ to 2 significant figures.

Likewise, $652.73 = 653$ to 3 significant figures, and $652.73 = 653.7$ to 4 significant figures.

Thus, the phrase 'significant figures (s.f.)' refers to the place values, starting from the left-most non-zero digit, to which importance is attached in order to give an approximate value of a number.

Example 7.13

Write the following correct to the number of significant figures given in brackets.

(a) 546.52 (3) (b) 546.52 (4)
(c) 8.029 6 (1) (d) 8.029 6 (2)
(e) 0.009 25 (1) (f) 997 375 (3)

Solution

We start counting the significant figures from the first non-zero digit at the left of the number.

(a) 546.52 = 547 to 3 s.f.

(b) 546.52 = 546.5 to 4 s.f.

(c) 8.029 6 = 8 to 1 s.f.

(d) 8.029 6 =.8.0 to 2 s.f. (In this case, the zero must be given after the decimal point; it is significant.)

(e) 0.009 25 = 0.009 to 1 s.f. (9 is the first non-zero digit. The two zeros after the decimal point are not significant figures. However, they must be written down to keep the place value correct.)

(f) 997 375 = 997 000 to 3 s.f. (The last 3 zeros must be written down to keep the place value correct.)

Note:

1. If the next digit after the last significant digit is 4 or less, the number is rounded down. If it is 5 or more, the number is rounded up.
2. Zeros that lie between non-zero digits are significant.

e.g. 0.008 700 2

non significant — significant

Example 7.14

Write 0.081 043 correct to 5,4,3,2,1

(a) decimal places

(b) significant figures

Solution

Table 7.6 shows the decimal approximations.

approximations	*d.p.*	*approximations*	*s.f.*
0.081 04	*5*	*0.081 043*	*5*
0.081 0	*4*	*0.081 04*	*4*
0.081	*3*	*0.081 0*	*3*
0.08	*2*	*0.081*	*2*
0.1	*1*	*0.08*	*1*

Table 7-6

Exercise 7.8

1. Copy and complete Table 7-7.

	Number	Number of d.p.			Number of s.f.			Nearest whole number
		3	2	1	3	2	1	
	0.043 5	0.044	0.04	0.0	0.043 5	0.044	0.04	0
(a)	0.023 45							
(b)	0.005 130 7							
(c)	0.082 056							
(d)	6.893 4							
(e)	4 .624 7							

Table 7-7

2. Express the following correct to one decimal place.
   (a) 3.43 (b) 5.28 (c) 6.48
   (d) 5.46 (e) 8.37

3. Write the following correct to 3 significant figures.
   (a) 19.045 (b) 43.588 (c) 31.52
   (d) 0.034 56 (e) 0.054 6

4. State the number of (i) decimal places (ii) significant figures in:
   (a) 0.23 (b) 0.1 (c) 18.000 6
   (d) 0.565 (e) 0.006 5 (f) 33.076

5. Evaluate the following giving your answers correct to (i) 3 decimal places (ii) 3 significant figures.
   (a) 15.43 ÷ 0.8 (b) 11.78 ÷ 0.6
   (c) 12.47 ÷ 0.03 (d) 105.7 ÷ 2.45
   (f) 613.2 ÷ 3.75

Changing fractions to decimals

To change a fraction to a decimal,

(i) divide the numerator by the denominator to one more than the required number of decimal places and round

off as may be necessary, **or**

(ii) express as an equivalent fraction whose denominator is a power of 10, then write the fraction in decimal form.

Example 7.15

Write $\frac{3}{8}$ as a decimal.

Solution

$$\begin{array}{r} 0.375 \\ 8\overline{)\,3.000} \\ -24 \\ \hline 60 \\ 56 \\ \hline 40 \\ 40 \end{array}$$

or $\frac{3}{8} = \frac{3 \times 12.5}{8 \times 12.5}$

$= \frac{37.5}{100} = \frac{375}{1\,000}$

$= 0.375$

$\therefore = 0.375$

$\frac{3}{8}$ gives an exact decimal. We say that 0.375 is a **terminating** decimal.

Now, consider the fraction $\frac{2}{3}$.

$$\begin{array}{r} 0.66\ldots \\ 3\overline{)\,2.0} \\ -18 \\ \hline 20 \\ -18 \\ \hline 20 \end{array}$$

$\therefore \frac{2}{3} = 0.666\ldots$

In this case, $\frac{2}{3}$ gives a **non-terminating** decimal, 0.666...

Such a decimal is called a **recurring** decimal because certain digits keep on repeating.

Other examples of recurring decimals are:

2.545 454... 0.712 712... etc

A recurring decimal is written in short with a dot (.) above the recurring digit, e.g.

$0.333\ldots = 0.\dot{3}$

If more than one digit recurs, dots are put only above the first and last digits of the recurring group, e.g.

$25.535\,3\ldots = 25.5\dot{3}\dot{3}$,

$2.623\,623\ldots = 2.6\dot{2}\dot{3}$, etc

Sometimes, instead of dots, a line (called a '**vinculum**') is drawn over the recurring digits, e.g. $2.623\,623\ldots = 2.6\overline{23}$.

There are some non-terminating decimals which are not recurring, e.g.

3.141 592 654... (This number is known as 'pi', written as π),

2.718 2... (This number is known as 'Planck's constant' and is denoted as e).

Exercise 7.9

Express each of the following as a decimal.

1. (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{3}{5}$ (d) $\frac{1}{10}$ (e) $\frac{1}{8}$

2. (a) $\frac{6}{10}$ (b) $\frac{3}{4}$ (c) $\frac{4}{5}$ (d) $\frac{7}{8}$ (e) $\frac{1}{25}$

3. (a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{9}$ (d) $\frac{5}{11}$ (e) $\frac{5}{27}$

4. (a) $4\frac{13}{20}$ (b) $7\frac{9}{16}$ (c) $\frac{29}{32}$ (d) $\frac{8}{13}$ (e) $3\frac{7}{23}$

Changing decimals to fractions

Most decimals can be expressed as fractions whose denominators are powers of 10. For example:

0.7 means seven tenths, written as $\frac{7}{10}$.

0.75 means seven tenths 5 hundredths, written as

$$\frac{7}{10} + \frac{5}{100} = \frac{7 \times 10}{10 \times 10} + \frac{5}{100}$$

$$= \frac{70}{100} + \frac{5}{100} = \frac{75 \div 25}{100 \div 25} = \frac{3}{4}$$

0.125 means one tenth two hundredths and 5 thousandths written as

$$= \frac{1 \times 100}{10 \times 100} + \frac{2 \times 10}{100 \times 10} + \frac{5}{1\,000}$$

$$= \frac{100}{1\,000} + \frac{20}{1\,000} + \frac{5}{1\,000}$$

$$= \frac{100+20+5}{1\,000} = \frac{125}{1\,000} = \frac{5}{40} = \frac{1}{8}$$

The place value of the last digit at the right of the number suggests the denominator of the fraction.

Example 7.16

Express 0.84 as a fraction in its simplest terms.

Solution

$0.84 = \frac{84 \div 4}{100 \div 4} = \frac{21}{25}$

Recurring decimals can be expressed as fractions as illustrated in Examples 7.17 to 7.19.

Example 7.17

Express $0.\dot{4}$ as a fraction.

Solution

Step 1: $10 \times 0.\dot{4} = 10 \times 0.444... = 4.444...$

Step 2: $1 \times 0.\dot{4} = 1 \times 0.444... = 0.444...$

Step 3:

$$\begin{array}{rr} 10 \times 0.\dot{4} & \\ -\,1 \times 0.\dot{4} & \\ \hline 9 \times 0.\dot{4} & \end{array} \quad \text{and} \quad \begin{array}{r} 4.444... \\ -\,0.444... \\ \hline 4.000... \end{array}$$

$\therefore \quad 9 \times 0.\dot{4} = 4$

Step 4: $0.\dot{4} = \frac{4}{9}$ *(Dividing both sides by 9)*

Thus, $0.\dot{4} = \frac{4}{9}$.

Example 7.18

Express $0.\dot{2}\dot{7}$ as a fraction.

Solution

Step 1: $100 \times 0.\dot{2}\dot{7} = 27.\dot{2}\dot{7}$

Step 2: $1 \times 0.\dot{2}\dot{7} = 0.\dot{2}\dot{7}$

Step 3:

$$\begin{array}{rr} 100 \times 0.\dot{2}\dot{7} & \\ -\,1 \times 0.\dot{2}\dot{7} & \\ \hline 99 \times 0.\dot{2}\dot{7} & \end{array} \quad \text{and} \quad \begin{array}{r} 27.\dot{2}\dot{7} \\ -\,0.\dot{2}\dot{7} \\ \hline 27.00 \end{array}$$

$\therefore \quad 99 \times 0.\dot{2}\dot{7} = 27$

Thus, $0.\dot{2}\dot{7} = \frac{27}{99} = \frac{3}{11}$.

Example 7.19

Convert $7.3\dot{6}$ to a fraction.

Solution

Step 1: $10 \times 7.3\dot{6} = 73.666...$

Step 2: $1 \times 7.3\dot{6} = 7.366...$

Step 3:

$$\begin{array}{rr} 10 \times 7.3\dot{6} & \\ -\,1 \times 7.3\dot{6} & \\ \hline 9 \times 7.3\dot{6} & \end{array} \quad \text{and} \quad \begin{array}{r} 73.666... \\ -\,7.366... \\ \hline 66.300 \end{array}$$

$\therefore \quad 9 \times 7.3\dot{6} = 66.3$

Step 4: $7.3\dot{6} = \frac{66.3}{9}$ *(Dividing both sides by 9)*

Thus $7.3\dot{6} = \frac{66.3 \times 10}{9 \times 10}$ *(To make numerator a whole number)*

$= \frac{663}{90} = 7\frac{11}{30}$.

Exercise 7.10

1. Write the following decimals (i) in words (ii) as fractions.
   (a) 0.7 (b) 0.21
   (c) 5.108 (d) 34.203

2. Write the following decimals as fractions.
   (a) 0.3 (b) 0.25
   (c) 0.007 (d) 0.28
   (e) 0.085 (f) 0.365

3. Write the following as mixed numbers in the simplest form possible.
   (a) 2.6 (b) 3.4
   (c) 5.004 (d) 23.12

4. Express the following as fractions in their simplest forms.
   (a) $0.\dot{3}$ (b) $0.\dot{6}$
   (c) $0.\dot{2}$ (d) $3.\dot{3}\dot{6}$
   (e) $0.2\dot{3}$ (f) $2.\dot{6}$
   (g) $0.\dot{1}8\dot{5}$ (h) $1.\dot{3}\dot{1}\dot{3}$

5. Write the following as (i) improper

fractions (ii) mixed numbers in the simplest form.

(a) 2.4 (b) 5.04

(c) 3.25 (d) $1.\dot{4}\dot{5}$

(e) $3.\dot{0}1\dot{5}$ (f) $4.\dot{4}\dot{2}$

(g) $5.2\dot{6}$ (h) $5.\dot{2}\dot{7}$

Problems involving decimals

This exercise emphasises the use of decimals in real life situations.

Exercise 7.11

1. A rectangular flower garden measures 38.47 m by 15.75 m. Calculate the total distance round the flower garden.
2. On a certain day, the highest reading on a barometer was 762.45 mm and the lowest was 758.66 mm. What is the difference between these readings?
3. A watering can holds 2.54 litres of water. How many such cans could be filled from a tank that holds 100 litres? How much water would be left over?
4. A triangular plot of land has sides of length 22.4 m, 31.5 m and 42.8 m. Find the total distance around the plot.
5. A piece of wire is 18.4 cm long. If 7.4 cm is cut from it, what length of wire remains?
6. For seven consecutive days, the meteorological department recorded the following amounts of rainfall: 0.5, 2.2, 0.9, 0.4, 1.6, 0.8 and 1.5 cm. Find the total rainfall for the week, giving your answer to the nearest whole number.
7. The product of two numbers is 1.340 3. If one of the numbers is 0.13, find the other number.
8. How many whole pieces of string 9.7 cm long can be cut from 1 500 cm of string?